MATHEMATICAL MODELS OF GAS FIRED BOILERS

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ABSTRACT

As for traditional boilers, the cyclical efficiency of a condensing boiler can be defined by its experimental efficiency at full load, for a given return water temperature and by its experimental stand-by losses, considering not only the boiler but its actual regulation system too. Measurements of natural gas consumptions during 8 years will be presented and used to simulate conventional and condensing boilers in domestic heating plants. To calculate seasonal efficiency of a boiler, a sinusoidal variation of loads during the season, from zero to a maximum value, can be assumed.

Keywords: gas, boiler, condensing, efficiency

1. NOMENCLATURE

AREA	surface of flat	[m^2]
days	= 1, 2, .. day0 during a season	[-]
days0	total days in a season	[-]
GAS	gas consumption during time	[m^3]
GAScons	gas from first day	[m^3]
GASy	total gas in a heating season	[m^3]
H_i	high heating value of the fuel	[J/ m^3]
LU	load	[-]
LUmax	maximum load	[-]
m	mean seasonal load	[-]
P_{cons}	input of the boiler	[kW]
P_{fl}	flue gas losses at partial load	[kW]
P_{fl0}	flue gas losses at full load	[kW]
PN	nominal output	[kW]
PR	experimental stand-by losses	[kW]
PR_X	stand-by losses at null load	[kW]
P_{sl}	surface losses at partial load	[kW]
PU	partial output of the boiler	[kW]
P_{vl}	ventilation losses at partial load	[kW]
P_{vl0}	ventilation losses at null load	[kW]
T	time-step	[hour]
TM	outlet water temperature	[°C]
TR	return water temperature	[°C]
TR*	nominal return water temp.	[°C]
TRp	design water temperature	[°C]
TR_y	water temperature at full load	[°C]
η^*	nominal efficiency at full load	[-]
η_0	experimental efficiency at full load	[-]
η_b	combustion efficiency	[-]
η_c	cyclical efficiency at partial load	[-]
η_{dc}	efficiency of distribution system	[-]
η_{ec}	cyclical efficiency of emitters	[-]
η_g	seasonal efficiency	[-]
η_{gc}	efficiency of boiler and regul. syst,	[-]

2. INTRODUCTION

Approximately 40% of final energy is consumed in buildings in the whole European Union (Eurostat 2007). Much attention is paid to the improved energy efficiency in building sector during the last years, because the sector harbours a considerable potential of primary energy saving and reduction of emissions, having a negative impact on the environment.

A large proportion of the European old housing stock consists of family houses, very often with a central boiler and no temperature control at the individual apartment level. Very often refurbishment takes place, involving replacement of the boiler (usually gas), incorporating central thermostatic control or thermostatic valves, while the pipes and radiators remain the same. It is very difficult to predict fuel consumption, control behaviour of the system and quality of the indoor environment (temperature amplitudes, etc.).

In order to build energy new efficient buildings, correct decisions have to be made already at the design stage. Heating system is among the systems, which are responsible for the consumption of the major part of energy in cold climate countries. It means that the design of this system can influence the overall energy performance of the building.

The system designer faces the challenge to select a suitable boiler in terms of lightweight or cast-iron, condensing or non-condensing, originally specified capacity (i.e. oversized) or re-calculated, etc. Some of the factors that should be considered include: boiler type, design temperatures, capacity of the system, control of the system (local, central, or both), thermal inertia of building and heating plant.

Requirements to evaluate the impact on environment of the energy-using products during its whole life cycle are set in directive 2005/32/EC of the European Parliament and of the Council of 6 July 2005 establishing a framework for the setting of eco-design requirements for energy-using products and amending Council Directive 92/42/EEC and Directives 96/57/EC and 2000/55/EC of the European Parliament and of the Council. Directive 2006/32/EC of the European

Parliament and of the Council of 5 April 2006 on energy end-use efficiency and energy services and repealing Council Directive 93/76/EEC recommends to use minimised life cycle cost analysis in particular cases (European Commission 1997).

A number of studies about and energy systems of building life cycle have been conducted. Some of them deal with the building construction and energy systems' life cycle. Some of them analyse energy systems (Prek 2004).

Analysis shows that in all cases the operation stage is most important, and there the biggest energy consumption occurs. That means that it is not so important in this case what materials or elements are used in the system, much more important is the efficiency of the heat generator.

The processes occurring in apartment heating can be analyzed and simulated by computer programs. These programs examine various situations in order to evaluate heat losses and seasonal fuel consumption.

Every gas-heating system includes a gas-fired boiler, a distribution system, a set of emitters and a control device of temperatures in the apartment. During a time-step *T,* the energy balance between energy contributions from emitters and the fuel consumption GAS is:

$$\text{Energy from heat emitters} = \text{GAS} \bullet H_i \bullet \eta_c \qquad (1)$$

where H_i is the high heating value of the gas, and η_c is the average efficiency of the boiler and the distribution system during the same time-step *T*.

These so-called cyclical efficiency η_c may be considered the product of the three cyclical efficiencies which may be attributed to the three subsystems in which the heating plant can be divided (Rosa and Tosato 1985):

$$\eta_c = \eta_{gc} \bullet \eta_{ec} \bullet \eta_{dc} \qquad (2)$$

where η_{gc} = cyclical efficiency of the boiler and its regulation system; η_{dc}=cyclical efficiency of the distribution system; η_{ec}=cyclical efficiency of the emitters.

The boiler (conventional and condensing) is taken into account ($\eta_{dc} = \eta_{ec} = 1$) in this paper. However, as the regulation system affects the temperatures of the boiler and its efficiency, it is more correct to examine a boiler together with its regulation system η_{gc} instead of the boiler alone.

3. CONDENSING BOILER

The main differences between conventional systems and condensing boilers are:

- Condensing boiler technology requires that the exhaust gases are removed with *fans*, since, because of the strong cooling of the exhaust gases, their buoyancy is generally not sufficient.
- The exhaust gas pipes have to be *insensitive to humidity* because exhaust gases contain rests of humidity which condense in the exhaust gas pipes.
- *Condensate* has to be removed.
- The parts of condensing boilers, of the exhaust gas removal system and of the condensate removal system have to be *corrosion* resistant.
- The *second heat exchanger* was one of the first special constructional features in the beginning of the condensing boiler technology development. In order to use the heat energy of the exhaust gases an additional heat exchanger was added to a conventional boiler. Today such a heat exchanger is still used for large high-performance boilers.

4. MODEL OF BOILER OPERATING AT PARTIAL LOAD

There are numerous energy losses from a boiler. While most of them are minor, and comprise about 1% or less of fuel input, there are two or three major losses that typically represent 10 to 20 % of fuel input.

As described by Natural Resources Canada's Office of Energy Efficiency (OEE) (2006), Flue gas losses represent the heat in the flue gas that is lost to the atmosphere upon entering the stack. Stack losses depend on fuel composition, firing conditions and flue gas temperature. They are the total of two types of losses:

- Dry Flue Gas Losses: the (sensible) heat energy in the flue gas due to the flue gas temperature;
- Flue Gas Loss due to Moisture – the (latent) energy in the steam in the flue gas stream due to the water produced by the combustion reaction being vaporized from the high flue gas temperature.

Radiation and convection losses are independent of the fuel being fired in a boiler and represent heat lost to the surroundings from the warm surfaces of a high-temperature water generator. These losses depend mainly on the size of the equipment (e.g., small boilers have a proportionately larger percentage loss than large boilers), and the actual output relative to the maximum design output.

Considering a model 1, the major energy losses associated with boilers fall into three categories: flue gas losses to the chimney Pfl during the ON time, radiation and convection losses of external surfaces Psl during the ON and OFF times, ventilation losses to the chimney during the OFF time.

For condensing boilers it is very difficult to evaluate these losses by experimental tests at full load, at partial loads and at null load.

I prefer to consider for condensing and traditional boilers a different model (Model 2), more near to the experimental possibilities in a test rig (Rosa and Tosato 1988) .

A linear relationship between energy consumed and load, and steady-state model were considered as appropriate to calculate the energy consumption over a long period as during the winter period.

This linear law, referred to a unit of time and written in an adimentional form, is:

$$P_{cons}/PN = A + B \bullet PU/PN \qquad (3)$$

where P_{cons} = input of the boiler; PN = nominal output of the boiler; PU = partial output of the boiler during the operation; A, B = two constants.

This law is acceptable if the mean temperature of the water in the boiler is kept constant and is more or less valid for traditional boilers connected to a plant by a control mixing valve. As far as condensing boilers are considered, the temperature of the return water varies and a linear relationship similar to eqn. (3) is acceptable only for a narrow range of loads. For the whole load range the variation of flue gas losses P_{fl}, ventilation losses P_{vl} and surface losses P_{sl} have to be taken into account.

The author analyses conventional boilers connected to a plant with constant temperatures of water when the load varies. For this type of boilers he suggests that equation 4 should be used, where, differently from equation 3, the functions A and B (eq. 5 and 6) are not constant . Losses P_{fl}, P_{vl} and P_{sl} depend by a linear equation on loads and can be assessed by experimental tests at full load (i.e. P_{fl0}=0.2 and P_{sl0}=0.1) and at null load (i.e. P_{vl0}=0.3). The results of this simulation (Model 1) are plotted in fig. 1.

A second model (Model 2) of losses in a boiler is more near to experimental tests. The major energy losses associated with boilers fall into two categories:

- flue gas losses to the chimney during the ON time, radiation and convection losses of external surfaces Psl during the ON and OFF times, ventilation losses to the chimney during the OFF time.

As indicated in [2], a numerical model of a boiler can be defined by its experimental efficiency at full load η_0, at different return water temperatures TR, and by its experimental stand-by losses PR. It can be assumed that the operation of a boiler with an ON-OFF cyclical mode is based on the following assumptions:

- during the ON-time, the flue gas losses and the surface losses, as well as the efficiency, are the same as during the full-load operation;
- during the OFF-time, the efficiency is zero, and the ventilation and surface losses are a function of the mean temperature (TR + TM)/2 of the water, where TM is the outlet water temperature.

For condensing boilers and for conventional boilers too, the author considers stand-by losses PR at null load and steady efficiency η_0 at full load with constant temperatures of water when the load varies (Model 2). He suggests for them that equation 7 should be used, where the functions A and B (eq. 9 and 10) are not constants. Losses PR can be assessed by experimental tests as a function of mean water temperature in the boiler at null load. The results of this simulation are plotted in fig. 2 for condensing boilers (Model 2) and in fig.3 for traditional boilers (Model 2). The results of fig. 3 are the same of fig. 1.

The results of this simulation for condensing boilers with negligible stand-by losses PR and return water temperature TR proportional to the load are plotted in fig. 4 (Model 2).

If the boiler temperature (TR + TM)/2 is taken as constant and the losses $P_{vl} = P_{vl0} \bullet (1 - PU/PN)$ and $P_{fl} = P_{fl0} \bullet PU/PN$ are proportional to PU, according to Fig. 1, then:

$$\begin{aligned} P_{cons}/PN &= 1/\eta_c \bullet PU/PN \qquad (4)\\ &= P_{fl}/PN + P_{vl}/PN + P_{sl}/PN + PU/PN\\ &= P_{fl0}/PN \bullet PU/PN + P_{vl0}/PN \bullet (1-PU/PN) +\\ &\quad + P_{sl0}/PN + PU/PN\\ &= (1/\eta_b - 1) \bullet (1+P_{sl0}/PN) \bullet PU/PN + P_{vl0}/PN \bullet (1-PU/PN) +\\ &\quad + P_{sl0}/PN + PU/PN\\ &= A + B \bullet PU/PN \end{aligned}$$

where η_b = combustion efficiency, and

$$A = P_{vl0}/PN + P_{sl0}/PN \qquad (5)$$

$$\begin{aligned} B &= 1 + P_{fl0}/PN - P_{vl0}/PN \qquad (6)\\ &= 1 + (1/\eta_b - 1) \bullet (1+P_{sl0}/PN) - P_{vl0}/PN \end{aligned}$$

The experimental evaluation of η_b of traditional boilers doesn't present any problem and the Siegert formula ($\eta_b = 1 - C \bullet (T_f - T_a)/CO_2$) is generally used in laboratory and field tests. However, this evaluation doesn't take into account the condensing possibilities and refers to the low heat value. The calculation of P_{sl0} and P_{vl0} losses needs a monitoring rig usually employed only in a laboratory. These difficulties lead to the choice of another equation of energy balance

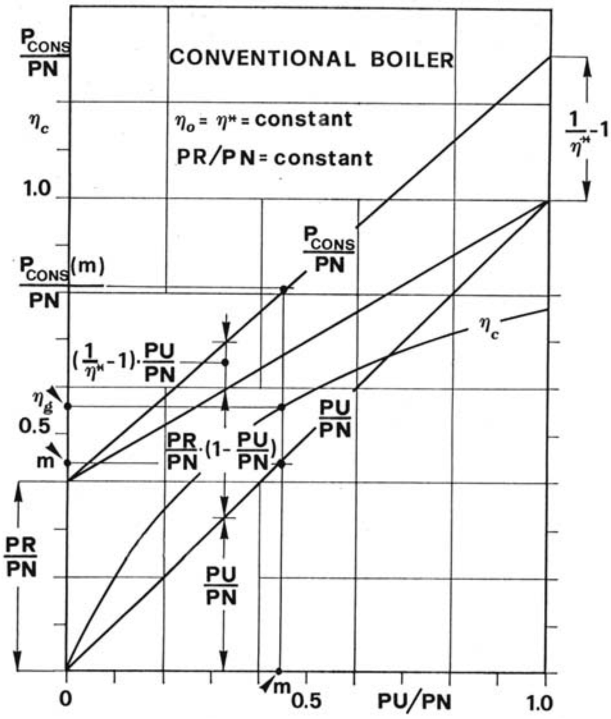

Figure 1. Acceptable model of a boiler at partial load when temperatures are more or less constant, using P_{fl}, P_{vl} and P_{sl} losses. (Model 1)

Accordig to Fig. 2, then it follows that:

$$
\begin{aligned}
P_{cons}/PN &= 1/\eta_c \bullet PU/PN \qquad (7)\\
&= PR/PN \bullet (1-\eta^*/\eta_0 \bullet PU/PN)\\
&\quad + (1/\eta_0 - 1) \bullet PU/PN + PU/PN\\
&= PU/(PN \bullet \eta_0) + PR/PN \bullet (1-\eta^*/\eta_0 \bullet PU/PN)
\end{aligned}
$$

where η^* is the nominal efficiency at full load, corresponding to the nominal return water temperature TR* (usually 60 °C).

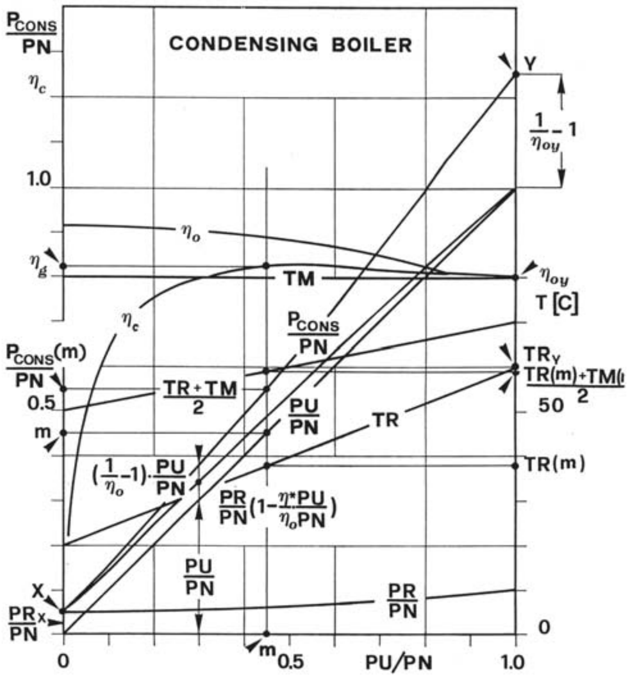

Figure 2. Model of boiler at partial load, using stand-by losses and efficiency η_0 at full load, considering their variation with water temperatures. Condensing boiler connected to plant/system by a 3-way mixing valve.

As represented in Fig. 3, if $\eta_0 = \eta^*$, η_0 and PR are constant (as in a boiler at constant water temperature), the linear relationship, eqn. (4), for the cyclical efficiency, is:

$$
\begin{aligned}
P_{cons}/PN &= 1/\eta_c \bullet PU/PN\\
&= PR/PN + (1/\eta^* - PR/PN) \bullet PU/PN\\
&= A + B \bullet PU/PN \qquad (8)
\end{aligned}
$$

where, according to eqns. (5) and (6):

$$
\begin{aligned}
A &= PR/PN = P_{vl0}/PN + P_{sl0}/PN \qquad (9a)\\
B &= 1/\eta^* - PR/PN \qquad (9b)\\
&= 1 + P_{fl0}/PN - P_{vl0}/PN\\
&= l/\eta_b \bullet (1 + P_{sl0}/PN) - P_{sl0}/PN - P_{vl0}/PN
\end{aligned}
$$

$$1/\eta^* = 1/\eta_b \bullet (1 + P_{sl0}/PN) \qquad (10)$$

This result is not accurate for condensing boilers, because these present a difference between η_0 and η^* which can be larger than 15%. Two kinds of condensing boilers can be considered depending on whether PR/PN is negligible or not. For condensing boilers having a very low level of stand-by losses PR/PN=0, eqn. (7) leads to:

$$\eta_c = \eta_0 \qquad (11)$$

As represented in Fig. 4, this efficiency is not a linear relationship because η_0 varies with the return water temperature.

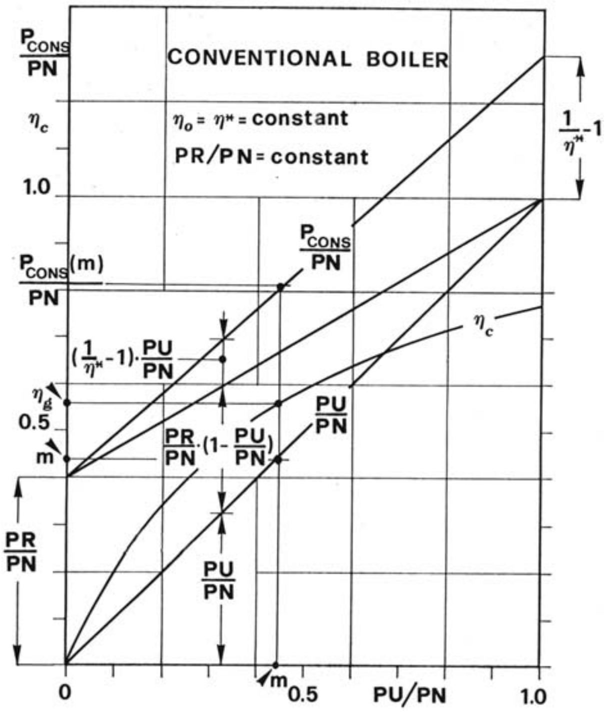

Figure 3. Acceptable model of a boiler at partial load; acceptable when temperatures are more or less constant, using stand-by losses PR and steady efficiency η_0

Not all condensing boilers have negligible stand-by losses, especially when the evacuation of flue gases takes place through the natural draught. This occurs only if the flue gas channels are very long and characterized by low pressure drops. For these boilers the relationship used was that of eqn. (7), which, obviously, is not linear and it depends on η_0, PR and the dependence law of the boiler temperatures TR and TM from load PU/PN (determined by the regulation system).

The P_{cons}/PN curve in the case of a condensing boiler connected to the plant by a 3-way mixing valve is represented in Fig. 2, where, again, the relationship is not linear

If the PR curve is unknown, the cyclical efficiency by eqn. (7) cannot be directly calculated. A straight line from the losses at zero load (point X) to the energy consumption at full load (point Y) can be assumed and the values of the constants A and B are:

$$A = PR_X/PN \qquad (12)$$

$$B = 1/\eta_{0y} - PR_X/PN \qquad (13)$$

and the linear relationship is:

$$\begin{aligned} P_{cons}/PN &= 1/\eta_c \cdot PU/PN \qquad (14)\\ &= PR_X/PN + (1/\eta_{0y} - PR_X/PN)\cdot PU/PN\\ &= PR_X/PN\cdot(1-PU/PN+1/\eta_{0y}\cdot PU/PN) \end{aligned}$$

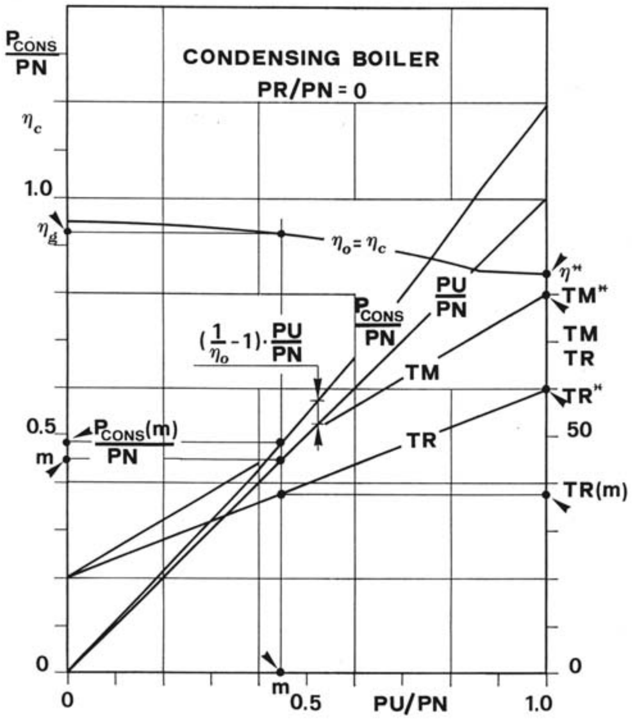

Figure 4. Model, using stand-by losses PR and steady efficiency η_0 of a condensing boiler without stand-by losses, directly connected to a plant having TR_d=TR*.

The loss PR_X/PN at PU/PN=0 has to be evaluated at boiler temperature controlled by the regulation system at this load (for direct connection PR_X/PN=0). Values of P_{cons}/PN at the point Y can be obtained from laboratory. The efficiency η_{0y} at full load assumes the η_0 value corresponding to the return water temperature TR_y controlled by the system (the design return water TRy=TRp): $\eta_{0y}=\eta^*$ only if TRy=TRp =TR*.

5. SEASONAL EFFICIENCY

The amount of natural gas consumption, in an old apartment rented to students, was collected during an eight-year period.

This apartment measures 100 m^2 (AREA), is located in a multi-unit building and heated by an independent boiler (high efficiency boiler, 22 kW).

The students who lived in the apartment changed over the years. Their presence during the week was not constant (5 or 7 days) and their choice of the room temperature (18-22 °C) by the heating system control was different too.

The gas consumption data analysed are those going from September to June.

The total annual consumption GASy = (828 - 2250) m^3 and the profile of GAS consumption, beginning from September, are shown in Fig. 5.

The variation of GAS during the season is evident and the normalized total consumption per floor area GASy/AREA is very variable between 8.2 and 22.5 m^3/m^2.

Fig. 6 describes the adimentional values of GAScons/GASy, which is variable only during the season and can be represented by a simple sinusoidal equation (15).

Its derivative is the load LU=PU/PN of the boiler. As a result, during a season, maximum loads LUmax of the boiler are required in a short period during the winter and LUmax values can be much lower than 100 %. In the other months LU can be assumed to vary from zero to LUmax with a cosinus law (16).

GAScons/GASy =
= 50*(1+sin(days - days0/2)*6,28/365/1,3))[%] (15)

LU/LUmax=
=100*cos((days-day0/2)*6,28/365/1,3)[%] (16)

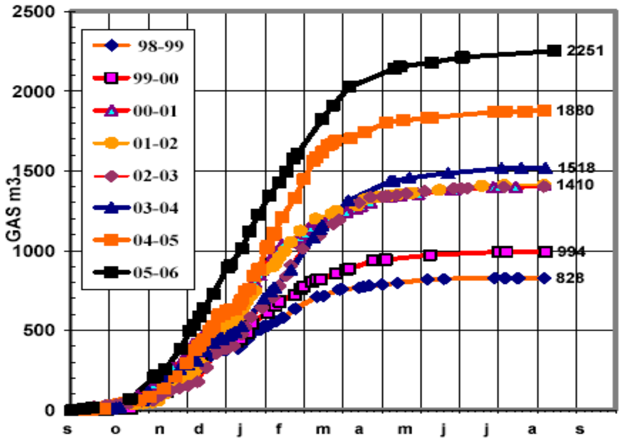

Figure 5. Profile of consumption of GAS

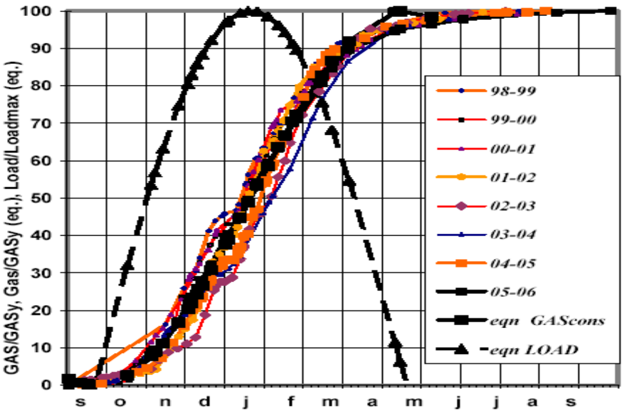

Figure 6. Profile of adimentional of GAS/GASy and LU/LUmax.

The seasonal efficiency η_g may be assumed to be equal to the cyclical efficiency η_c calculated at the mean seasonal value *m* of the loads LU = PU/PN only if the efficiency is constant:

$$\eta_g = \eta_c(m) \tag{17}$$

In order to predict the seasonal efficiency of a boiler η_g an accurate result can be gained by using the following procedure:

(1) two experimental curves representing the boiler efficiency at full load and losses must be determined by laboratory tests as function of water temperatures:

η_0(TR) and

PR((TR+TM)/2).

These two curves do not depend upon the regulation system;

(2) establish both the return TR and the delivery TM temperatures as function of the loads LU in relation to the adopted regulation system;

(3) construct the curve of the cyclic efficiency η_c as function of loads LU;

(4) use N+1 loads from 0, 1/(N)*LUmax, 2/(N)*LUmax, … to LUmax. As an example, given N=4: (0, 0.25, 0.5, 0.75, and 1)*LUmax, calculate the corresponding values of cyclic efficiencies for the boiler;

(5) consider a sinusoidal variation of loads during the season from zero to LUmax and calculate N intervals of days corresponding to the same loads: T1, T2, T3 and T4;

(6) Use the equation (18)

$$\eta g = \frac{\sum_{i=0}^{N-1} \eta_c \cdot LU \cdot T + \sum_{i=1}^{N} \eta_c \cdot LU \cdot T}{\sum_{i=0}^{N-1} LU \cdot T + \sum_{i=1}^{N} LU \cdot T}. \tag{18}$$

6. CONCLUSIONS

Sinusoidal loads from zero to maximum load (less than 100%) during a season seem to be acceptable in apartment heating plants. The relationship between energy consumption end energy demand in cyclical operation can be used with a sufficient degree of precision to calculate the seasonal consumption as a function of the sinusoidal load during the season.

A relationship of the cyclic efficiency of the boiler and of the regulation system can be calculated using two experimental curves: the efficiency at full load and the stand-by losses.

This relationship can be used for condensing boilers and for other kinds of boilers, and for the usual regulation systems as soon as the appropriate values of the constants have been experimentally defined. The evaluation of the seasonal efficiency of a boiler requires the knowledge of the efficiency at some loads.

By comparing the seasonal efficiencies it can be assumed that:

- The seasonal efficiency of a condensing boiler can be higher by 25% more than the efficiency of traditional boilers.
- The seasonal efficiencies of condensing boilers increase as the load decreases, while in the same condition the efficiencies of traditional boilers decrease.
- When the seasonal maximum load of the boiler, LUmax, is low, the seasonal efficiency of a condensing boiler presents a further increase, while that of traditional boilers decreases.
- SO_x, NO_x, dust and soot, etc., which are the constituents of the flue gas, can be dissolved in the condensed water, and the pollutants emitted to the environment can be noticeably reduced.
- It is of great significance both to environmental protection and energy saving to utilize condensing boilers.

ACKNOWLEDGMENTS

This work has been supported by Italian Ministero Istruzione Università Ricerca MIUR funds. Their contribution is gratefully acknowledged.

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