HYBRID APPROACH FOR MODELING AND CONTROL WAREHOUSE SYSTEMS

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ABSTRACT

The aim of this paper is twofold: firstly we define a dynamical model for representing warehouse's product flows, taking into account both customer demands and stocked quantities. Secondly we introduce an automatic controller regulating products handling into the inventory structure, which aims to guarantee the satisfaction of customer demands as well as maintaining safety stocks. The proposed work is based on hybrid approach for dynamical system definition and control, developed using dedicated informatics tools. This tool is developed to improve optimal control for real systems, such as car traction control and chemical processes. In this paper we use this tool to approach a warehouse management problem improving an optimal control algorithm.

Keywords: warehouse management, hybrid systems, model predictive control (MPC).

1. INTRODUCTION

The interest about operation management is continuously rising, this trend gives impulse to new approaches to manage different enterprise aspects aiming to optimize the performance of the whole production cycle and increase competitiveness. In this contest techniques developed to optimize the production using planning and control techniques (PPC) may be applied (Vollmann and Barry 1997).

Recent studies focus on a new approach for PPC problem based on application of automatic control theory, defining a new problem formulation called Automatic Production Control (APC). In Wiendahl and Breithaupt (1998) the production process is modelled as a dynamical system which describes the material flows through the plant, given an analytic representation of internal dynamics in several work stations. The main issue of this approach is to define a controller connected to the model, which allows the automatic control of the production system. This technique aims to having an output flow of finished goods which satisfies customer demand, controlling at the same time works in progress into the plant.

Recently supply chain concept has been introduced in operations management: in this context the attention is extended on all transformation phases of material rows in finished goods, integrating logistic, production and information management. This approach consolidates the concept of material flow through the different phases of transformation process, including support logistic (internal and external), production phases and sale organization. According to APC problem formulation in literature several dynamical models to control supply chain are presented. These models are joined by chain structure to evidence the material flow through different resources which compose the modelled supply chain. This representation may be found also in models of production processes, where each block represents a particular work phase in terms of work in progress and product queues.

In Boccadoro and Martinelli (2006) a model of supply chain composed by a series of working sites is proposed, connecting the sites by downstream flow of materials and upstream flow of information. The control law is developed for satisfying the demand of final client synchronizing exchange of information and material within the chain. Using a linear discrete dynamical system to represent supply chain it's possible to implement optimal control laws using LQ or H-infinity approach.

In Dumbar and Desa (2005) a constrained model for supply chain introducing variable boundaries in a discrete linear model is developed, Mixed integer problem is solved to control the proposed system. This approach is more complex than the analytic formalization proposed in Boccadoro and Martinelli (2006), but allows to adopt a more complex and robust control law, named Model Predictive Control (MPC). This control theory appear to be a very powerful instrument also used to control hybrid systems, as in Balduzzi and Menga (2000) where an hybrid formulation for manufacturing production control is developed.

This paper focuses on controlling an inventory system which consists of two different stock areas crossed by flow materials. The model follows hybrid linear system specification which allows to determine a optimum control strategy for material flows. This strategy aims to minimize backlog orders satisfying the customer demand, also trying to maintain an acceptable safety stocks level. Hybrid formulation allows us to introduce logical values and constrains in the system dynamic making it more realistic. Using a dedicated toolbox to create the hybrid models and control them it's possible to modify and analyze the performance of close loop system; in the same way by setting the parameters of MPC law it's possible to test different warehouse configuration and working logics.

2. PROBLEM FORMULATION

For simplicity of comprehension we start modelling a single product class warehouse (i.e. inventory is made of an unique product); extension to more than one product classes is then proposed.

It may be assumed that the layout of the warehouse system is chain-like (Figure 1), where two different zones are highlighted, one called *SE* (service zone) where the input material is stocked for the replenishment in other zone, called high density zone (*HD)* which represents the warehouse structure, where products are picked for satisfying the customer demand.

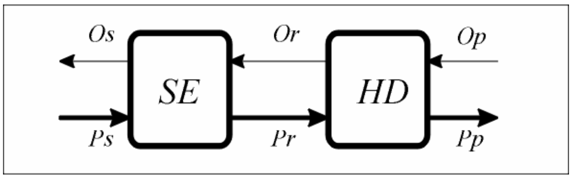

Figure 1: Warehouse layout with information flow (solid lines) and material flow (thick lines).

The *SE* and *HD* zones interact exchanging information and material, in particular the information is represented by orders: external as the customer demands or internal as the replenishment orders.

- *Op* : Product Order (customer demand)
- *Or* : Replenishment Order
- *Os* : Suppliers Order

The material flow is modelled using three variables: supplying products which enter in the system, replaced products in *HD* zone and output products picked out from the warehouse:

- *Ps* : Products from suppliers
- *Pr* : Replaced products
- *Pp* : Picked products

Warehouse dynamics are described using state equations of discrete time linear systems, where the state evolves in discrete steps, clocked by a sampling time *Ts*. The value of *Ts* is also important, because it must be compatible with data variability. Too small *Ts* raises the complexity of calculation without increasing extracted information from data, too big *Ts* reduces the sensibility of the system losing important dynamical behaviours. By using discrete notation it is possible to take advantage of the high potential of this approach, with a wide literature on modelling and controlling discrete systems, also with powerful informatics instruments for analysis and implementation. In this work we decided to represent a generic time step with *k* and the next with *k+1*, thus omitting *Ts*.

To define a correct model some hypothesis are needed regarding relations between order information and product quantities into the system. We assume that replenishment orders at generic time *k* are satisfied at *k+1*, as well as the supplier orders are satisfied with a known delay *τ*. Under these hypothesis we can describe the dynamics of product stocks in the high density and service zone using the following equations:

$$\begin{cases} HD(k+1) = HD(k) + Pr(k) - Pp(k) \\ SE(k+1) = SE(k) + Ps(k-t) - Pr(k) \end{cases} \qquad (1)$$

New stocks in high density zone *HD(k+1)* will be equal to the quantity *HD(k)* plus replenishment products *Pr* minus the product picked out from warehouse *Pp* at the generic *k* interval. In the same way service products *SE* are increased by the products from suppliers *Ps*, and decreased by the replenishment *Pr*.

We can introduce a picking strategy related to customer demand, considering another dynamic taking into account backlog orders:

$$BL(k+1) = BL(k) + Op(k) - Pp(k) \qquad (2)$$

The backlog amount *BL* depends on its previous value, customer demand *Op* and quantities delivered from the warehouse *Pp*.

The aim of this work is to implement a control algorithm that allows us to manage warehouse dynamics (1)(2) trying of obtain a material flow through the warehouse which optimizes its performance.

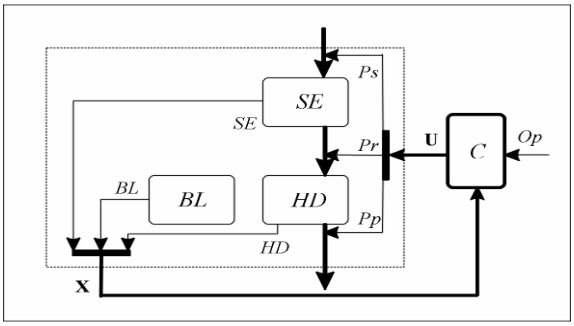

Figure 2: Control problem structure, with warehouse model (dashed block) and controller (solid block).

The control action may be applied choosing appropriate values for endogenous input of the system $\mathbf{u} = [Pr\ Ps\ Pp]^T$, depending on system state $\mathbf{x} = [HD\ SE\ BL]^T$ and exogenous input *Op* (Figure 2).

Considering a generic *k* step, given the states $\mathbf{x}(k)$ and the demand *Op(k)* we calculate the inputs $\mathbf{u}(k)$ which forces the future states $\mathbf{x}(k+1)$ to tend at the reference values $\mathbf{r}$. The control law is determined by minimization of performance index:

$$\mathbf{J}(\mathbf{x},\mathbf{u}) = \|\mathbf{Q}(\mathbf{x}-\mathbf{r})\| + \|\mathbf{Pu}\| \qquad (3)$$

3. HYBRID MODEL

3.1. Dynamics

The model is generalized for a generic product class i, considering the dynamics and suppliers behaviour independent for each class, while constrains regard all classes depending on the warehouse structure and throughput capability.

Following the approach illustrated in the previous section we define a system state $\mathbf{x}_i = [BL_i \; HD_i \; SE_i]^T$, where:

- BL_i : Backlog orders
- HD_i : High density area stocks level
- SE_i : Service level area stocks level

an input array $\mathbf{u}_i = [o_i \; p_i \; r_i \; s_i]^T$ with:

- o_i : Customers demand
- p_i : Picked products
- r_i : Replaced products
- s_i : Product from suppliers

and system output $\mathbf{y}_i = [p_i]$. These variables are used to write the difference equations which characterize the system behaviour:

$$S : \begin{cases} BL_i(k+1) = BL_i(k) + o_i(k) - p_i(k) \\ HD_i(k+1) = HD_i(k) + r_i(k) - p_i(k) \\ SE_i(k+1) = SE_i(k) + s_i(k-\tau_i) - r_i(k) \\ y(k) = p_i(k) \end{cases} \quad (4)$$

The following figure illustrates a graphical form for the system described by (4):

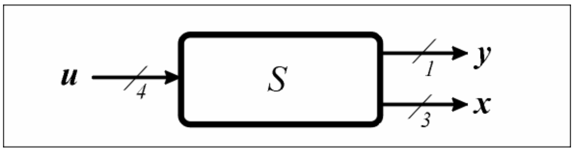

Figure 3 : Graphical representation of warehousing system S described by (4).

3.2. Constrains

The present approach for hybrid model structure follows the framework defined in Bemporad and Morari (1999). This work provides an useful approach for hybrid systems formulation, providing mathematical tools for studying them. This formulation allows us to add constrains for system variables, obtaining a theoretic model which simulates the behaviour of a real system.

The introduction of constraints about capacity or maximum throughput of warehouse is implemented with inequality on all P product class variables. We identify 3 types of constrains: structural, logistic and operative.

Structural constrains consist in the total quantities of products stored into different warehouse zones, at each step k these values can't exceed the maximum capability of each area, HD_{max} e SE_{max} :

$$\sum_{i=1}^{P} HD_i(k) \leq HD_{max} \qquad \sum_{i=1}^{P} SE_i(k) \leq SE_{max} \quad (5)$$

We consider that each product class has its own supplier, with given lead time and lot size specifications. For this reason logistic constrains to define a fixed lot size supplies and the delay of τ_i steps are introduced. Lot sizes Ls_i are inserted into the model as parameters:

$$s_i(k) \in [0, Ls_i] \qquad \forall i \in [1:P] \quad (6)$$

The input s_i can assume only one of two values at each time step k, according the fact that supplier of i product is activated or not. For modeling this supplying phase it's necessary to introduce a logical variable used to activate the supplier shipment, also an auxiliary real variable is needed to represent the effective product quantities introduced into the system. The introduction of logical inputs inside a continuous dynamics makes the model hybrid thus imposing the use of dedicated tools.

Operative constrains simulate the effective throughput capability of warehouse in terms of product units moving during a step, called the maximum throughput TP_{max} :

$$\sum_{i=1}^{P} r_i(k) + \sum_{i=1}^{P} p_i(k) \leq TP_{max} \quad (7)$$

3.3. Mixed Logical Dynamical models

How showed in Bemporad an Morari (1999) it' s possible to define hybrid dynamics with physical laws, logical rules and constrains. An important result is the demonstration that difference equations, logics rules and inequalities can be written in an uniform form by using Mixed Integer Linear Inequalities containing continuous and logical variables. For clarity we introduced the distinction between a continuous part containing variables and equations which describe temporal dynamic, and logical part composed by boolean variables and logical rules. The connections between the two parts and an uniform representation are implemented with the introduction of auxiliary variables (continuous and logical) which allow us to write a good representation for the whole system; but on the other hand it increases the model dimensions, making control law calculation more complex.

Proposed model for warehouse system composed from dynamic (4) and constrains (5)(6)(7) could be represented by a Mixed Logical Dynamical model (MLD model) using linear equations and inequalities :

$$\begin{cases} \mathbf{x}(k+1) = A\mathbf{x}(k) + B_1\mathbf{u}(k) + B_2\boldsymbol{\sigma}(k) + B_3\mathbf{z}(k) \\ \mathbf{y}(k) = C\mathbf{x}(k) + D_1\mathbf{u}(k) + D_2\boldsymbol{\sigma}(k) + D_3\mathbf{z}(k) \\ E_2\boldsymbol{\sigma}(k) + E_3\mathbf{z}(k) \leq E_1\mathbf{u}(k) + E_4\mathbf{x}(k) + E_5 \end{cases} \quad (8)$$

the state vector, input and output contain logical and continuous variables, $\mathbf{x}(k) = [\mathbf{x}_c\ \mathbf{x}_l]$, $\mathbf{u}(k) = [\mathbf{u}_c\ \mathbf{u}_l]$, $\mathbf{y}(k) = [\mathbf{y}_c\ \mathbf{y}_l]$. The vector **z** contains auxiliary continuous variables, while **σ** contains logical ones.

The proposed model is characterized by a state with 9 continuous components :

$$\mathbf{x} = [BL_1\ HD_1\ SE_1\ BL_2\ HD_2\ SE_2\ BL_3\ HD_3\ SE_3] \quad (9)$$

representing backlog orders, products stocked in high density zone and in service zone for each class. The input is composed by 6 continuous components and 3 logical ones:

$$\mathbf{u} = [p_1\ r_1\ p_2\ r_2\ p_3\ r_3\ s_1\ s_2\ s_3] \quad (10)$$

representing picked and replaced quantities (*p* and *r*) and the boolean variable which activates the supplier orders (*s*) for each product class. Also we need to introduce 3 auxiliary continuous variables to convert the logical value of *s* to fixed lot quantities.

The introduction of constrains and complex dynamics exponentially increases the problem complexity, raising the number of dependent variables and the dimension of the MLD model (8). A correct analysis to reduce dynamics and constrains used for modelling every system is needed, to limit the model size and make the proposed method feasible.

4. OPTIMAL CONTROL

4.1. Feedback Structure

The aim of this method is the implementation of an algorithm which controls material flows for the model defined in the previous section. The problem is approached as a state regulation for dynamical systems, where customer demands are assimilated to state perturbations, and the controller operates forcing the state around a reference.

Focusing on a generic product class representation (Figure 3) it' s possible to structure the control problem following the next scheme:

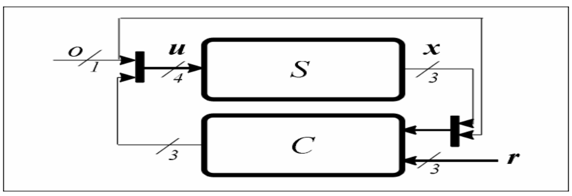

Figure 4: Control scheme with feedback connection between system *S* and controller *C*.

Figure 4 shows the feedback connection between system S and controller C. For a generic step *k* the controller C has the state system **x***(k)* and customer demand *o(k)* as inputs. C calculates the **u***(k)* signals for controlling future state **x***(k+1)* toward the constant reference **r**.

4.2. Model Predictive Control

To define control signals we used the Model Predictive Control law (MPC) that proved to be a good tool for working with hybrid systems (Bemporad and Morari 2001). This technique needs a dynamical model for the predictive approach, using a MLD model it's possible to predict the future behaviour of the system and calculate control signals. The predictions are used to estimate the dynamics of the system during a finite period in the future (prediction horizon of *T* steps), starting from current state. The resolution of optimal control problem with MPC approach give, at generic step *k*, a sequence of *T* input signals **U***(k) which drives the state around a reference:

$$\mathbf{U}^*(k) = [\mathbf{u}^*(k)\ \ \mathbf{u}^*(k+1)\ \ ...\ \ \mathbf{u}^*(k+T-1)] \quad (11)$$

Only the first sample **u****(k)* of this sequence is applied to the model, discarding the others. At the next step the algorithm is replicated starting from new state observations. The receding horizon approach allow us to share the action control over several steps, satisfying the model constrains.

Optimal input sequence (11) is calculated solving a Mixed Integer Linear Programming problem (MILP), where a functional cost is minimized under constrain of a MLD dynamic. In this case to determine MPC law Mixed Integer Predictive Control problem is solved, where a performance index is minimized to obtain an optimal control sequence:

$$\mathbf{U}^*(k) = \underset{\mathbf{U}(k)}{arg\ min}\, \mathbf{J}(\mathbf{x}(k), \mathbf{U}(k)) \quad (12)$$

In the cost function **J** predicted quantities are used, where **x***(k+i|k)* represents the state estimation at step *k+i* calculated from observations at time *k* using MLD model:

$$\mathbf{J}(\mathbf{x}(k), \mathbf{U}(k)) = \sum_{i=0}^{T-1} \left\| \mathbf{x}(k+i\,|\,k) - \mathbf{r} \right\|_{\mathbf{Q}} + \left\| \mathbf{u}(k+i) \right\|_{\mathbf{P}} \quad (13)$$

This methods allow us to define an optimal control sequence for the steps between *k* and *k+T-1*.

4.3. Performance Index

Aim of this work is to adapt the illustrated control approach to modelling inventory management strategies, where the total cost function (13) can be seen as an overall performance indicator for the system. The control law is implemented to define product quantities exchanged between different areas of warehouse (10).

Considering the model (8) containing the dynamics defined in (4), it's possible to create a performance index which uses the product quantities in *SE* and *HD* zones, with particular attention to backlog orders as measure of inventory management efficiency. The quantities of products into the warehouse zones are compared to given references in order to satisfy safety stocks in the warehouse, whereas backlog orders are forced to zero to satisfy customer demands. The differences between state and references are introduced into the performance index:

$$\mathbf{J}(\mathbf{x}(k),\mathbf{U}(k)) = \sum_{i=0}^{T-1} \left\| \mathbf{x}(k+i \mid k) - \mathbf{r} \right\|_{\mathbf{Q}} \qquad (14)$$

where $\mathbf{x}$ is the predicted state of the system, in term of backlog orders and products stocked in the zones (9).

The reference vector contains the constant expected values for each state components with zero for backlog orders and safety stock level for service and high density zones:

$$\mathbf{r} = [0\ RH_1\ RS_1\ 0\ RH_2\ RS_2\ 0\ RH_3\ RS_3] \qquad (15)$$

$\mathbf{Q}$ is a diagonal matrix which contains the weight of every cost component, choosing appropriate values for each component it's possible to characterize the behavior of the system, in this case we penalize backlog orders more than product level into zones. The performance index (14) is computed using infinity norm, which extracts the maximum values from a weighted vectors, as example a generic vector $\mathbf{v}$ are considered:

$$\left\| \mathbf{v} \right\|_{\mathbf{Q}} = \max_i \left| (\mathbf{Q}\mathbf{v})_{\mathbf{i}} \right| \qquad (16)$$

where $(\mathbf{Qv})_i$ is the *i* components of row-column product between the $\mathbf{Q}$ matrix and vector $\mathbf{v}$.

5. IMPLEMENTATION

5.1. Hysdel Language

Several representations for hybrid systems are proposed in literature (Heemels and Schutter 2001), this large number depends on a different application field which requires a dedicated form for hybrid systems. In this section is shortly illustrated the structure of Discrete Hybrid Automata (DHA), where a series of continuous dynamics influence the state switches into a finite state automata. The DHA is composed by four subparts:

Finite State Machine (FSM): that contains the discrete part of system, composed by discrete state and logical rules which sets the current state.

Switch Affine System (SAS): part containing the continuous part of the system, it's structured as a series of continuous dynamics which are selected by a dedicated index.

Event Generator (EG): this subpart allows to translate continuous information into discrete ones with generation of events, these events are generated from the analysis of continuous state in SAS.

Mode Selector (MS): it's the counterpart of EG, where discrete signals from FSM are elaborated for selecting the active dynamic in SAS.

The tool used in this paper for hybrid system definition is the HYSDEL (HYbrid Systems DEfinition Language) (Torrisi and Bemporad 2004), this application allows us to write a text representation for dynamical behaviour of systems in a well posed syntax using DHA representation.

First part of HYSDEL model is the interface, where the model structure is defined, with numeric parameters, states, inputs and outputs. Developed warehouse model works with 3 product classes each of them with dedicated suppliers. Each supplier has a known lead time, we consider class 1 is supplied with lead time of one step, class 2 with lead time 2 and third class has 3 steps of lead time. Section relative to third product class is reported below:

```
INTERFACE  {
    PARAMETER {
               REAL HD_min;
               REAL HD_max;
               REAL SE_min;
               REAL SE_max;
               REAL BL_min;
               REAL TP_max;

               REAL Ls_3;
               }
    STATE     {
               REAL BL_3 [-1e3,1e3];
               REAL HD_3 [-1e3,1e3];
               REAL SE_3 [-1e3,1e3];
               REAL se3_d1 [-1e3,1e3];
               REAL se3_d2 [-1e3,1e3];
              }
    INPUT     {
               REAL o_3 [-1e3,1e3];
               REAL p_3 [-1e3,1e3];
               REAL r_3 [-1e3,1e3];
               BOOL s_3;
              }
    OUTPUT    {
               REAL y_3;
               }
```

We can note that the input and state vectors are the same of system (4), in the state are present of two auxiliary states (*se3_d1*, *se3_d2*) needed to introduce delay for modelling supplier lead time. In interface section parameters necessary for constrain definitions related to (5)(6)(7) are defined.

Second section of HYSDEL file contains the dynamic implementation, with the definition of auxiliary variables to represent complex behaviours, and difference equations for continuous dynamics of system:

```
IMPLEMENTATION {
    AUX    {
            REAL z_s3;
            }
    DA      {
        z_s3 = {IF s_3 THEN Ls_3 ELSE 0};
            }
     CONTINUOUS
            {
            BL_3 = BL_3 + o_3 - p_3;
            HD_3 = HD_3 + r_3 - p_3;
            SE_3 = SE_3 + se3_d1 - r_3;
            se3_d1 = se3_d2;
            se3_d2= z_s3;
            }
     OUTPUT
            {
            y_3 = p_3;
            }
```

The DA section containing the expression to convert the logical input *s_3* to a product quantity, using the system parameter *Ls_3* and the auxiliary real variable *z_s3*.

The last section is used to define constrains on system variables:

```
MUST   {
        HD_1+HD_2+HD_3 <= HD_max;
        HD_1+HD_2+HD_3 >= HD_min;

        SE_1+SE_2 +SE_3<= SE_max;
        SE_1+SE_2 +SE_3>= SE_min;

        BL_3 >= BL_min;

        p_1+p_2+p_3+r_1+r_2+r_3<= TP_max;

        p_3>=0;
        r_3>=0;

        HD_3>=0;
        SE_3>=0;
        BL_3>=0;
        }
```

Used tools allows us to convert HYSDEL model in some different representations for hybrid systems, this is possible thanks to the result in Heemels and Schutter (2001), where the equivalence between different representations is showed. In this case the HYSDEL model is converted in a MLD model. The report of this operation is showed:

```
MLD  hybrid  model  generated  from  the
HYSDEL file <wh_system.hys>

12 states  (12 continuous, 0 binary)
12 inputs  (9 continuous, 3 binary)
3 outputs  (3 continuous, 0 binary)

3 continuous auxiliary variables
1 binary auxiliary variables
31 mixed-integer linear inequalities
```

5.2. MPC toolbox

Control of warehouse model is implemented with a Matlab toolbox presented in Bemporad (2004), which allows to apply a MPC law on hybrid systems. This instrument contains a command that builds a MILP problem starting from a MLD model and the parameters necessary for defining a performance index. The parameters of this function are the state components that must appear in performance index, the references for this components and coefficients for weighting each difference between state value and references. In the proposed approach only the system state contributes to the cost function (14) whose components which describe system dynamics (9), these components are weighted with a 9x9 diagonal matrix **Q**.

The MILP solution gives us a controller generating a number of signals equal to the number system input, in this model we present 3 exogenous inputs (product demands) which influence the dynamic but can't be manipulated. To avoid this problem the control law is calculated on an extended model, where exogenous inputs are assimilated by state components: this solution allows us to obtain a control input with 9 components (10) for the warehouse model.

The prediction horizon is set to 3 steps, this values allow us to consider dynamical behaviours of the system, at the same time to reduce the calculation complexity with the reduction of MILP problem dimensions. Report of MILP formulation is showed below:

```
Hybrid  controller  based  on  MLD  model
wh_extended <wh_extended.hys> [Inf-norm]

 15 state measurement(s)
 0 output reference(s)
 0 input reference(s)
 9 state reference(s)
 0  reference(s)  on  auxiliary  continuous
z-variables

 42 optimization variable(s)
(30 continuous, 12 binary)
201 mixed-integer linear inequalities
sampling time = 1, MILP solver = 'glpk'
```

The entire state contains the real state of the system (9 components), 3 auxiliary states to model supplier lead time (one for class 2 and two for class 3) and 3 extended states for product demands. The auxiliary and extended components of the state are not forced with references, because don't give additional information about inventory behaviour.

After the controller definition it's possible to simulate the behaviour of controlled model using Simulink. This operative step needs the definition of simulation run length and the product demand for each class.

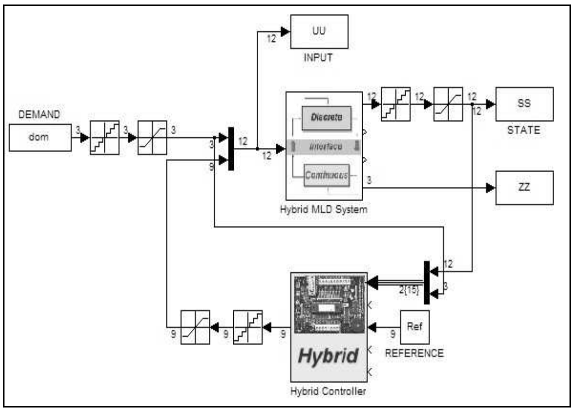

Figure 5: Simulink model to implement hybrid MPC on warehouse MLD model.

In the Simulink model (Figure 5) quantization blocks are introduced to guarantee integer quantities for state and input signals in the system. This expedient allow us to introduce more reality but increase uncertainty in the dynamics, changing the calculated quantities by the system and control blocks: this perturbation may be assimilated to a measurement noise. For this reason the infinity norm is used, to decrease the noise sensibility of closed loop system and at the same time increase the robustness of control actions.

6. NUMERICAL EXPERIMENTS

The control implementation and simulations of proposed model are developed using MATLAB 7.1 and Simulink 6.3 necessary to use the Hybrid Toolbox V 1.1.7 which requires the Model Predictive Control Toolbox.

Initially we tested the system using an impulse input for customer demands; this test highlights the behaviour of the system, showing the functioning in term of product quantities. The second test is developed to test the sensibility of the model changing the maximum throughput in HE zone.

The constant values of some parameters used during the tests are showed in Table 1:

Table 1 : Constant parameters used during the tests

	Class 1	Class 2	Class 3	Total
Ref. HD	100	150	200	450
Ref. SE	100	150	200	450
Lot Size	100	200	300	-
W_{BL}	20	20	20	-
W_{HD}	3	3	3	-
W_{SE}	1	1	1	-

For each class the reference quantities are reported, these values form the vector **r** in cost function (15), also it used as initial conditions for the simulations. The weight matrix **Q** is diagonal with non zero elements
$[\ W_{BL1}\ W_{HD1}\ W_{SE1}\ W_{BL2}\ W_{HD2}\ W_{SE2}\ W_{BL3}\ W_{HD3}\ W_{SE3}]$. The chosen values give the priority at demand satisfaction during the control action, successively safety stocks are restored.

6.1. Impulse Response

For this simulation the maximum throughput of HD zone is set to 300 units for period, and customer demand is a impulse at time 1 of 400 units for each class. This test put under stress the closed loop system that works for several steps to restore the initial conditions. For simplicity the sum of all class dynamics are proposed:

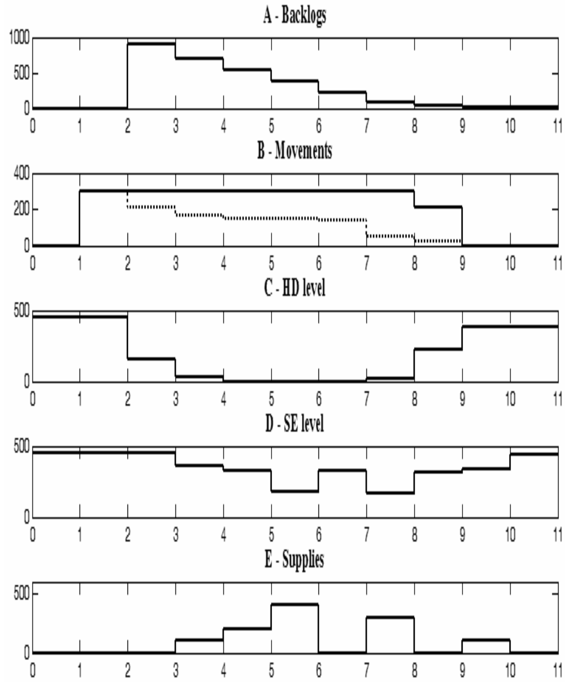

Figure 6: A - total backlog orders; B - total movement actions in HD zone (picking activity in dashed line); C - product level in HD zone; D - product level in SE zone; E - supply input of products.

The system reacts to the impulse with a step of delay for the strictly causal nature of dynamics, in Figure 6.A we can see the backlog increment at time 2. The advance reaction showed in Figure 6.B is a consequence of Simulink model (Figure 5), where the input signals are extracted with customer demands (block UU on the top) canceling the feedback delay in the graph.

Initially the controller forces the system on the picking action (dashed line in Figure 6.B) to reduce backlogs (Figure 6.A), but this behavior takes to zero the HD level (Figure 6.C), when HD zone becomes empty the replenishment quantity compensates the picking action (step 4 to 7), after that the controller increment replenishment input to restore the initial conditions. At the same time supply inputs (Figure 6.E) are set to guaranties safety stock in SE zone (Figure 6.D). The product quantities entered in the SE zone Figure 6.E) are in step form because sum of two level signals.

6.2. Throughput Analysis

This experiment aims at testing the sensibility of the proposed model changing crucial system parameters, the proposed analysis focus on maximum throughput. The movement capacity in HD zone is an important value which influences the model capability to satisfy customer demand through picking and replenishment actions.

To obtain comparable data the customer demand foe each class is maintained the same during all simulation tests :

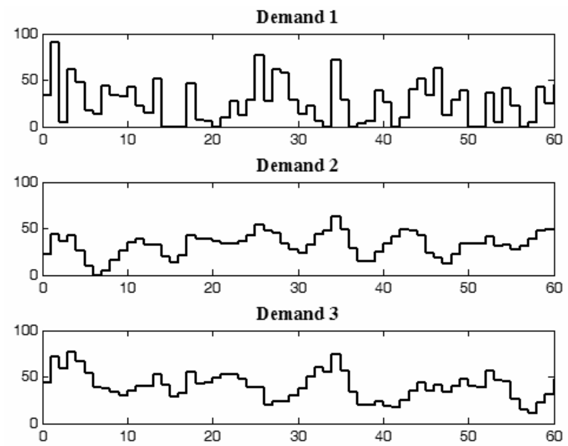

Figure 7: Demand profiles used in the throughput analysis

To evaluate the system performance we used two indexes. The first one taken into account is the percentage of backlog units over the period of simulation. This value is calculated for each class considering the ratio between the total backlog orders and total product demanded over the simulated period; with the same method the percentage of total backlogs is calculated. Second index evaluates the ratio between the handling operations in HD zone and the total handling operations capability.

These indexes are then evaluated in different simulation runs where the maximum throughput changes value between 200 and 400 units per period step. Results of these tests are showed in the next table:

Table 2: Results of throughput analysis

		Backlogs %				
	TP_m	Class 1	Class 2	Class 3	Total	Tp %
1	200	29,12	23,56	19,40	23,36	98,02
2	220	15,37	12,10	10,00	12,12	89,62
3	240	11,69	9,45	6,89	9,01	82,32
4	260	9,95	8,05	5,82	7,66	76,13
5	280	8,96	7,70	5,49	7,14	70,69
6	300	8,02	6,50	4,71	6,18	65,79
7	320	7,90	5,75	4,30	5,74	61,70
8	340	7,34	5,25	3,93	5,27	58,07
9	360	7,40	5,35	3,81	5,27	54,84
10	380	6,78	5,25	3,73	5,04	51,95
11	400	6,53	4,95	3,48	4,78	49,36

From this table we draw argue some interesting observation. With small value of throughput the warehouse performance is poor, the dynamics are conditioned by high level of backlog orders and saturation of movement capability. High values for maximum throughput assures good backlog performance but with a real low utilization of the handling capacity. The medium throughput gives good performance, balancing the backlog quantity and utilization percentage. Total backlogs and throughput percentages are plotted in Figures 8 and 9.

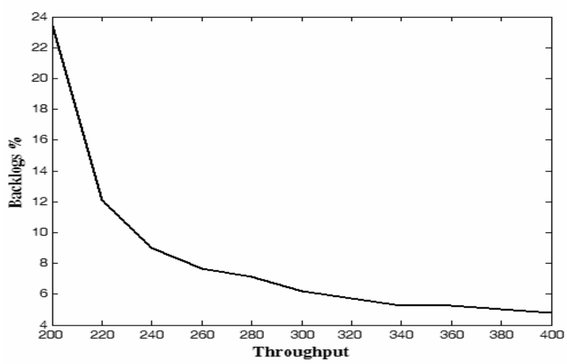

Figure 8: Percentage of total backlog units.

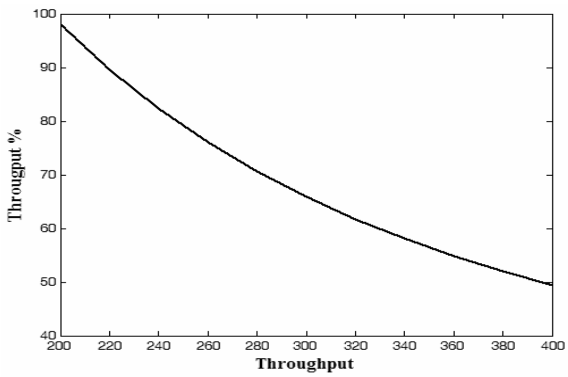

Figure 9: Percentage of capability utilization

The total results of simulation, with maximum throughput set to 300 units per period, are showed in Figure 10:

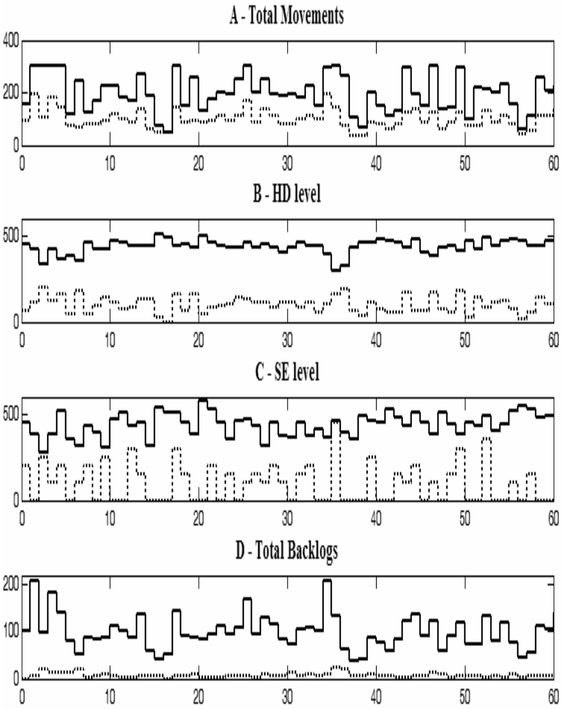

Figure 10: Total quantities of response for TP_max=300 units per period. A Total movements, picking activity in dotted line; B - HD level, replenishment activity in dotted line; C - SE level, supplies in dotted line; D - Total demand, backlog orders in dotted line.

7. CONCLUSIONS

In this paper a inventory management problem was defined using a two stages warehouse structure.

The dynamical model was developed using the hybrid system approach developed in Bemporad and Morari (1999). The HYSDEL language showed good potential to define linear hybrid system, combining easy syntax, useful code structure and powerful instruments to convert text expression in mathematical model (Torrisi and Bemporad 2004). The proposed model was characterized by different constrains for states and continuous input, moreover the use of boolean inputs allowed us to model fixed lot size supplies without significant increments of complexity. Backlog policy to manage customer demands was adopted to obtain realistic dynamics (Dumbar and Desa 2006). The decision to use a discrete time to model the warehouse, based on the availability of efficient tools, gives promising results; though a detailed analysis for parameter choice was necessary.

The MPC control policy was used to implement a inventory management strategy for the warehouse model. This algorithm was frequently adopted in control problems with constrained dynamics, in particular hybrid version (Bemporad and Morari 2001) showed high efficiency. The used toolbox (Bemporad 2004) allowed us to utilize the full flexibility of MPC law in term of cost function structure, tracking reference, weighted components and norm evaluation. The management policy adopted in this paper for inventory control gave good results about customer demand satisfactions and safety stock regulation.

The developed numerical experiments highlighted a acceptable behaviour of closed loop system, which proved to work following a logical criterion. Also the presented model demonstrates a good sensibility for parameter variations, this peculiarity increases the possibilities to use this approach in a real case study.

The proposed solution for inventory management may be extended in term of model complexity, adding logic dynamics to manage picking and replenishment phases for modelling different types of warehouse, also introducing different behaviours for product classes. The used toolbox can be used to implement very large and complex hybrid models and it give as well the instruments to analyze and control them.

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