DESIGNING PID CONTROLLER FOR 4TH ORDER SYSTEM BY MEANS OF ENHANCED PSO ALGORITHM WITH DISCRETE CHAOTIC DISSIPATIVE STANDARD MAP

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ABSTRACT

The main aim of this paper is the utilization of discrete chaotic Dissipative standard map based chaos number generator to enhance the performance of PSO algorithm. This paper presents application of proposed algorithm to design PID controller for 4th order system. Results of this application are compared with previously published results in the area of evolutionary PID controller design.

Keywords: PID controller, Optimization, Evolutionary Algorithms, PSO algorithm, Chaos, Dissipative standard map.

1. INTRODUCTION

In the past decades PID controllers became a fundamental part of many automatic systems. The successful design of PID controller was mostly based on deterministic methods involving complex mathematics. Recently (Nagraj et al., 2008) soft-computing methods were used with great results for solving the complex task of PID controller design.

Evolutionary algorithms are important part of softcomputing methods and one of them is particle swarm optimization algorithm (PSO). As proposed in (Davendra et al. 2010) using chaos number generator may improve the performance of an evolutionary algorithm for the task of PID controller design. This paper presents using of Dissipative standard map as a discrete chaotic system for the chaotic number generator and implementation of this chaotic generator into PSO algorithm. This enhanced PSO algorithm is applied on the PID controller design problem for 4th order system.

2. PARTICLE SWARM OPTIMIZATION ALGORITHM

PSO (Particle swarm optimization algorithm) is the evolutionary optimization algorithm based on the natural behaviorof bird and fish swarms and was introduced by Eberhart and Kennedy in 1995 (Kennedy, Eberhart 1995, Eberhart, Kennedy2001) as an alternative to genetic algorithms (Goldberg, David, 1989) and differential evolution (Storn, Price, 1997). Term "swarm intelligence" (Eberhart, Kennedy, 2001) refers to the capability of particle swarms to exhibit surprising

intelligent behavior assuming that some form of communication (even very primitive) can occur among the swarm particles (individuals).

In each generation, a new location of a particle is calculated based on its previous location and velocity (or "velocity vector"). One of PSO algorithm disadvantages is the rapid acceleration of particles which causes them to abandon the defined area of interest. For this reason, several modifications of PSO were introduced to handle with this problem. Within this research, chaos driven PSO strategy with inertia weight was used. Default values of all PSO parameters were chosen according to the recommendations given in (Kennedy, Eberhart 1995, Eberhart, Kennedy 2001). Inertia weight is designed to influence the velocity of each particle differently over the time (Nickabadi et al., 2011). In the beginning of the optimization process, the influence of inertia weight factor w is minimal. As the optimization continues, the value of w is decreasing, thus the velocity of each particle is decreasing, since w is the number < 1 and it multiplies previous velocity of particle in the process of velocity value calculation. Inertia weight new modification PSO strategy has two control parameters w_{start} and w_{ends} . New w for each generation is then given by Eq. 1, where *i* stand for current generation number and *n* for total number of generations.

$$w = w_{start} - \frac{\left(\left(w_{start} - w_{end}\right) * i\right)}{n}$$
(1)

Chaos driven random number generator is used in the main PSO formula (Eq. 2) that determines new "velocity", thus the position of each particle in next generation (or migration cycle).

$$v(t+1) = v(t) + c_1 \cdot Rand \cdot (pBest - x(t)) + c_2 \cdot Rand \cdot (gBest - x(t))$$
(2)

Where:

 $\begin{array}{l} v(t+1) - \text{New velocity of particle.} \\ v(t) - \text{Current velocity of particle.} \\ c1,c2 - \text{Priority factors.} \\ pBest - Best solution found by particle. \\ gBest - Best solution found in population. \\ x(t) - \text{Current position of particle.} \end{array}$

Rand – Random number, interval <0, 1>. Within Chaos PSO algorithm, the basic inbuilt computer (simulation software) random generator is replaced with chaotic generator (in this case, by using of Dissipative standard map).

New position of particle is then given by Eq. 3, where x(t+1) is the new position:

$$x(t+1) = x(t) + v(t+1)$$
(3)

3. DISSIPATIVE STANDARD MAP

The Dissipative Standard map is a two-dimensional chaotic map. The parameters used in this work are b = 0.1 and k = 8.8 as suggested in (Sprott 2003). The Dissipative standard map is given in Fig. 1. The map equations are given in Eq. 4 and 5.

$$X_{n+1} = X_n + Y_{n+1} (\text{mod}\,2\pi) \tag{4}$$

$$Y_{n+1} = bY_n + k\sin X_n \pmod{2\pi}$$
(5)



Fig. 1.Dissipative standard map

4. PROBLEM DESIGN

This section contains the description of the PID controller, used model of 4^{th} order system as well as the description cost functions (CF).

4.1. PID Controller and 4th order system

The PID controller contains three unique parts; proportional, integral and derivative controller (Astrom 2002). A simplified form in Laplace domain is given in Eq. 6.

$$G(s) = K \left(1 + \frac{1}{sT_i} + sT_d \right)$$
(6)

The PID form most suitable for analytical calculations is given in Eq.7.

$$G(s) = k_p + \frac{k_i}{s} + k_d s \tag{7}$$

The parameters are related to the standard form through: kp = K, ki = K/Ti and kd = KTd. Estimation of the combination of these three parameters that gives the lowest value of the four test criterions was the objective of this research.

The transfer function of used 4th order system is given by Eq. 8 (Davendra et. al. 2010)

$$G(s) = \frac{1}{s^4 + 6s^3 + 11s^2 + 6s}$$
(8)

4.2. Cost function

Test criterion measures properties of output transfer function and can indicate quality of regulation. Following four different integral criterions were used for comparison purposes: IAE (Integral Absolute Error), ITAE (Integral Time Absolute Error), ISE (Integral Square Error) and MSE (Mean Square Error). (Nagraj et al., 2008, Davendra et al., 2010).

These test criterions (given by Eq. 9-12) were minimized within the cost functions for the enhanced PSO algorithm.

1. Integral of Time multiplied by Absolute Error (ITAE)

$$I_{ITAE} = \int_{0}^{t} t |e(t)| dt$$
2. Integral of Absolute Magnitude of the Error (IAE)
(9)

$$I_{IAE} = \int_{0}^{t} \left| e(t) \right| dt \tag{10}$$

3. Integral of the Square of the Error (ISE)

$$I_{ISE} = \int_{0}^{\infty} e^{2}(t) dt$$
⁽¹¹⁾

4. Mean of the Square of the Error (MSE)

$$I_{MSE} = \frac{1}{n} \sum_{i=1}^{n} (e(t))^2$$
(12)

5. RESULTS

All experiments were focused on the optimization of the four different specification functions as given in previous section. The best results of the optimization with corresponding values of k_p , k_i and k_d together with selected response profile parameters are presented in Table 1.

When tuning a PID controller, generally the aim is to match some preconceived 'ideal' response profile for the closed loop system. The following response profiles are typical (Landau, 2006):

Overshoot: this is the magnitude by which the controlled 'variable swings' past the setpoint. 5 - 10% overshoot is normally acceptable for most loops.

Rise time: the time it takes for the process output to achieve the new desired value. One- third the dominant process time constant would be typical.

Settling time: the time it takes for the process output to die between, say +/-5% of setpoint.

From the statistical reasons, optimization for each criterion was repeated 30 times. The best results in all tables are highlighted by bold number. Results of the simple statistical comparison for the optimizations by means of chaos driven PSO algorithm are given in tables 2 and 3.

Optimized system responses are depicted in Figures 2a - 2d and compared in Figures 3 and 4.

Table 1: The best r	esults for 4 th	order system
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Criterion	CF	Кр	Ki	Kd	Overshoot	Rise Time	Settling time
IAE	12.347900	6.008590	0.007254	11.752400	0.153488	0.016000	0.048100
ITAE	15.533400	5.385930	0.000256	7.902410	0.102804	0.020500	0.034400
ISE	6.405160	5.194300	0.155542	20.815600	0.257472	0.011600	0.064300
MSE	0.032026	5.214940	0.156873	20.840000	0.258542	0.011600	0.064300

Table 2: Average steady state responses for 4th order system

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Criterion	Avg. overshoot	Avg. rise time	Avg. settling time
IAE	0.158410	0.015680	0.047500
ITAE	0.133150	0.018297	0.046077
ISE	0.257114	0.011603	0.064340
MSE	0.257894	0.011570	0.064293

Table 3: Statistical overview of the criterions (CF) values for 4th order system

Criterion	Max CF	Min CF	Avg. CF	Median	Std. dev.
IAE	12.614000	12.347900	12.418417	12.395900	0.072049
ITAE	21.534500	15.533400	17.626730	17.401250	1.594303
ISE	6.408370	6.405160	6.405957	6.405765	0.000841
MSE	0.032060	0.032026	0.032032	0.032029	0.000008

Results for PID controller design obtained by chaos driven PSO algorithm are compared in Table 4 with previously published result of evolutionary algorithms SOMA and DE with chaos implementation (Davendra et al., 2010) and non-heuristic Ziegler-Nichols method.

Table 4: Comparison of other methods and proposed enhanced PSO

Criterion	Z-N	DE Chaos	SOMA Chaos	PSO Chaos	
IAE	34.941300	12.330500	12.330500	12.347900	
ITAE	137.565000	15.384600	15.384600	15.533400	
ISE	17.842600	6.410260	6.410260	6.405160	
MSE	0.089213	0.032027	0.032027	0.032026	

From the presented results in Table 4, it follows that proposed enhanced PSO was surpassed by other two compared heuristic methods for the two test criterions and for other two it has given better results. Nonheuristic Ziegler-Nichols methods was outperformed in all four cases by all three compared evolutionary algorithms. System responses are compared in Figure 3. Detailed view on the beginning of simulation interval is depicted in Figure 4. As can be seen from these two Figures, the optimization based on the different criterions led to significantly different system responses; nevertheless the all designed PID controllers were able to quickly and precisely stabilize the system.



Figure 2: Optimized system responses for K_p, K_i and K_d obtained by four integral criterions.



Figure 3: Comparison of optimized system responses for K_p, K_i and K_d obtained by four integral criterions.



Figure 4: Detailed view on the optimized system responses for K_p, K_i and K_d obtained by four integral criterions.

CONCLUSION

This research was focused on the utilization of chaos driven PSO with discrete chaotic Dissipative standard map in the case of estimation of the PID controller optimal settings for 4th order system.

Presented data and graphical simulation outputs lend weight to the argument that implementation of chaotic Dissipative standard map as a random number generator into PSO algorithm may lead to satisfactory performance in the case of solving the problem of optimal PID controller design for 4th order system.

Future research will be aimed to the possibilities of the development and improvement of the enhanced chaos driven PSO algorithm to achieve better results and explore more possible applications for this promising optimization approach.

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