MODELING A WRONG MAINTENANCE POLICY

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ABSTRACT
The behavior of a single unit system, which is maintained according to the preventive policy, despite it fails with a constant failure rate, is focused.
A discrete event simulation (DES) enables to compare the expected number of failures that belong to alternative maintenance scenarios: the corrective and the preventive one.
The results comparison shows, at a first glance, that the preventive maintenance has to be preferred.
A deeper analysis, based on the information content registered at each step of a simulation process, on the Skellam function properties and on the Small Number Law, helps to clarify this strange behavior.
The real aim of modeling such a system would be to refuse the constant failure rate as an operations statement but only as a missing-information maintenance state. The simulation model enables to find more comprehensive information content about the behavior of stochastic single unit maintenance.

Keywords: Single item maintenance, discrete event simulation, Skellam function, small number Law, learning by simulation.

1. INTRODUCTION

Maintenance has ever been a critical issue for management of industrial processes. Since early sixties, for military purposes, maintenance literature was already extensive and rapidly growing. Barlow and Hunter (1960) and Barlow and Proschan (1967) started to evaluate the state probabilities of a complex system. Few years later, Jorgenson et al. (1966) edited a comprehensive report about optimal maintenance policies for stochastically failing equipment: the corrective, preventive and preparedness maintenance policies were defined as well the uselessness of use the preventive maintenance to items which fail according to constant failure rate if compared with the application of the corrective one. They also unified the principle optimal preventive models both for preventive and preparedness maintenance.

Henceforth preventive maintenance policy gained growing attention: economic models of optimization were performed over a horizon of thirty years and as a consequence new comprehensive focusing review about specific maintenance techniques have tried to make a picture of the state of the art (Sheut and Krajewski, 1994, Dekker 1996).

Due to the constant improvement of technology, processes have become more complex while service levels and most of all higher reliability performances are required. As a consequence the cost of preventive maintenance became the most important for industrial companies (Jardine et al. 2006).

Therefore, Researchers focused analysis on more efficient maintenance approaches such as condition-based maintenance (CBM) which are being implemented to handle the situation.

The state of art confirms the early structure of maintenance policies and their suggested applications: preventive policy is to be dedicated to monotone failure rate items; corrective maintenance is the only answer to random failure occurrence.

Relatively little has been written about the limits of corrective maintenance.

Inspired by this literature background, I was trying to compare the results of maintenance policies applied to a system during a simulation exercise designed for students in logistics course of master degree.

Because of the educational nature of the experiment, the application of above mentioned maintenance policies was applied to a simple case study such as that is represented by a single item system; just to make clear the contribution of proper maintenance policy to economic outcomes, it was simulated the application of corrective and preventive maintenance although the single item fails according to a constant failure rate.

The experiment was designed, however, so as to change the failure mode of the component so as to determine which strategy is better suited to the conditions change scenario.

The study below reported describes how the simulation modeling of a system can led to gain a better understanding of the focused problem and to overcome critical issues and maths tricks.

The study subtends the importance of *learning by simulating.*

The paper is structured as follows: a case study is described in order to define the problem; it regards the modelling of a single unit system; the model belongs to the discrete event simulation type: it is at the same time the creator and the solver of an apparent paradox. A brief discussion of the features of the model enables to solve the counter current behaviour and shows how is deep the information content of a simulation model.

2. THE CASE STUDY

25.01.2013 – Department of Industrial Engineering, University of Catania – During a maintenance lecture –

The topic of items reliability has just been introduced as the probability that an item could work without failing until a fixed time of mission is reached and under working conditions which are similar to that are declared by the supplier.

The general reliability equation, dedicated to constant failure rate items, was presented:

$R(t)=e^{-\lambda Tm}$

where λ is the constant failure rate and *Tm* is the time of mission.

In order to discuss the principle maintenance policies, the memory-less property was defined as a special feature of constant failure rate items.

The elegant application of the Bayesian theorem was introduced to consider the constant failure rate as the joint probability that the single item fails during the next elementary time interval, *dt*, if a failure didn't yet occur.

Given a number of items, N_0, which are subjected to a reliability test, at each control step the following equations can be written:

$N_0 = N_F(t)+N_S(t)$

$R(t) = N_S(t)/N_0$

$\lambda = dN_F(t)/dt\ l/N_S(t) = cost$

$\lambda\ dt = dN_F(t)/(dt\ N_0)\ N_0/N_S(t) = f(t)\ dt/R(t) = cost$

where $N_F(t)$ and $N_S(t)$ are respectively the number of failed and save units at each time *t*.

The memory-less property enables to understand how is fallacy to replace a constant failure rate item before it fails because, after the replacement, the failure rate doesn't change as the above mentioned joint probability suggests; so the corrective maintenance policy is the only one model which is to be taken in to account (Jorgenson et al. 1966).

A brief mathematics procedure was performed in order to compare the corrective and preventive maintenance results when they are applied to a constant failure rate single unit system.

So a single unit system was considered in order to define the conceptual maintenance model; it fails according to a constant failure rate, λ.

The corrective maintenance horizon can be evaluated by the following equation:

$$H = NF_{CM}\ MTTF \quad (1)$$

where $MTTF=\lambda^{-1}$ is the mean time to failure and NF_{CM} is the expected number of failures.

On the other hand, the preventive maintenance horizon leads to replace the single unit if it doesn't fail before the end of a fixed preventive maintenance period; the preventive maintenance horizon can be written as follows:

$$H = NF_{PM}MTTF'+ N_{PM}T_{PM} = NI_{PM}\ MTTI \quad (2)$$

where:

NF_{PM} is the number of failures which anyway are registered during the preventive scenario;

MTTF' is the mean time to failure of items which fail before the end of the preventive maintenance period T_{PM};

N_{PM} is the number of preventive maintenance replacements;

NI_{PM} is the expected number of interventions whether they belong to preventive or corrective type;

MTTI is the mean time to intervention along the preventive maintenance horizon.

The preventive maintenance features, as the mean time to failure, MTTF', the expected numbers of failures, NF_{PM}, and the expected number of preventive interventions, N_{PM}, can be evaluated by using the following further equations:

$$MTTF'=\int_0^{T_{PM}} tf(t)\ dt$$

$$MTTF'= MTTF-T_{PM}R(T_{PM})/(1-R(T_{PM})) \quad (3)$$

$$NF_{PM} = NI_{PM}(1- R(T_{PM})) \quad (4)$$

$$N_{PM} = NI_{PM}\ R(T_{PM}) \quad (5)$$

The substitution of equations (3), (4) and (5) in the equation (2) allows finding the equivalence between the above mentioned NF_{CM} and NF_{PM} numbers of failures:

$H = NF_{CM}\ MTTF =$

$= NI_{PM}\ (1- R(T_{PM}))\ [MTTF-T_{PM}R(T_{PM})/(1-R(T_{PM}))]+$

$+NI_{PM}\ R(T_{PM})\ T_{PM} = NI_{PM}\ [(1- R(T_{PM}))\ MTTF+$

$-T_{PM}\ R(T_{PM})+ R(T_{PM})\ T_{PM}] = NI_{PM}\ (1- R(T_{PM}))\ MTTF =$

$= NF_{PM}\ MTTF.$

To carve the latter equivalence on the stone, a little exercise was designed and showed to the students.

Figure 1 shows the scheme of corrective maintenance model which was presented to the students and its space state representation; the single unit can assume only a working or a failure state; the item fails with a constant failure rate $\lambda=0{,}01\ h^{-1}$, the defined maintenance horizon is H= 400 h.

The time to replace the failed item is considered deterministic and null ($\mu=\infty$).

The single item system was then modeled by sampling a sequence of items which simulate the system according to the Monte Carlo method; let i denote the i-th working item and T_i the time to failure which is randomly sampled by using the inverse function of the cumulated exponential probability density:

$T_i = -\ln(1-R_i)/\lambda$

where R_i is the i-th random number belonging to]0..1[range.

The model was coded by using an Excel Microsoft® spreadsheet during the lesson; the well known WYSIWYG property enables to show each step of the coding process and to display it on the dashboard: every student can see what is modeled and how.

So the corrective maintenance policy is modeled, by iteration, evaluating the series of sampled times to failure, T_i, until $\Sigma^k_{i=1}T_i<H$; when $\Sigma^k_{i=1}T_i>H$ the corrective maintenance scenario is fully simulated and the number of failures can be counted as NF_{CM} = k-1.

Table 1 shows the spreadsheet model which was coded.

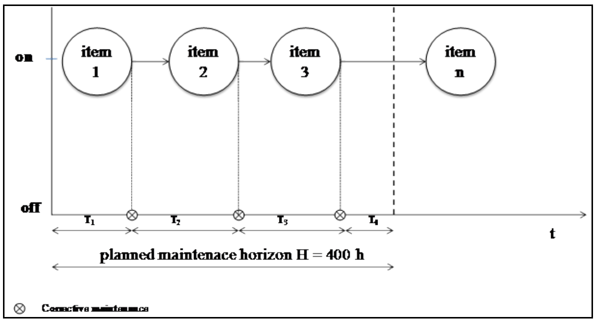

Figure 1: Single item corrective maintenance scheme

Table 1: Model of the corrective maintenance policy

Pos	Ri	T_i=-ln(1-R_i)/λ	$\Sigma^k_{i=1}T_i$	NF_{CM}
	[-]	[h]	[h]	[-]
1	0,93368139	271,33	271,33	1
2	0,49454915	68,23	339,56	2
3	0,54793473	79,39	418,95	3
4	0,46229076	62,04	418,95	3
5	0,00106236	0,11	418,95	3
6	0,14932840	16,17	418,95	3
7	0,23915160	27,33	418,95	3
8	0,95918196	319,86	418,95	3

The preventive maintenance scenario was simulated according to the age-dependent policy; the single unit is replaced at its age *t* or failure, whichever occurs first, where t=T_{PM} is the preventive maintenance period.

Figure 2 shows the scheme of the preventive maintenance model which was presented to the classroom and its space state representation.

Each simulated unit operates as follows:

$$T_i' = T_i \text{ if } T_i < T_{PM}, \text{ otherwise } T_i' = T_P \quad (6)$$

T_{PM} was also set equal to the item mean time to failure, MTTF = 1/λ.

The time to the preventive replacement is yet consider deterministic and null.

The preventive maintenance policy is modeled, by iteration, evaluating a series of random sampled time to failure items, T_i', which respects the equations set (6), until $\Sigma^k_{i=1}T_i'<H$; when $\Sigma^k_{i=1}T_i'>H$ the preventive maintenance scenario is fully simulated and the number of failures, NF_{PM}, can be counted among items for which both the following equation are respected:

$T_j'<T_{PM}$ and $\Sigma^j_{i=1}T_i'<H$.

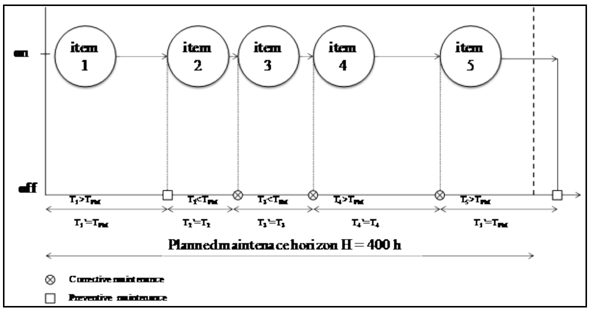

Figure 2: Single item preventive maintenance scheme

Table 2 shows the spreadsheet model which was coded.

Table 2: Preventive maintenance policy model

Pos	Ri	Ti	Ti'	$\Sigma^k_{i=1}T_i'$	NF_{PM}
		[h]	[h]	[h]	
1	0,93368139	271,33	100,00	100,00	0
2	0,49454915	68,23	68,23	168,23	1
3	0,54793473	79,39	79,39	247,62	2
4	0,46229076	62,04	62,04	309,67	3
5	0,00106236	0,11	0,11	309,77	4
6	0,14932840	16,17	16,17	325,95	5
7	0,23915160	27,33	27,33	353,28	6
8	0,95918196	319,86	100,00	453,28	6

To make more interesting the experiment it was evaluated the scenario in which the corrective cost was much more expensive than the preventive one; as a consequence, the maintenance cost function, both for corrective and preventive scenarios, depends only on the number of failures.

So the comparison of the two maintenance policies is based on the number of failures NF_{CM} and NF_{PM} in terms of average value and standard deviation.

The difference between NF_{CM} and NF_{PM} was also registered, for each simulation step, in order to calculate the relative distribution of frequency.

After few hundreds of replications the following results were discovered:

$f(NF_{PM} < NF_{CM}) \approx 40\%$;

$f(NF_{PM} = NF_{CM}) \approx 30\%$;

$f(NF_{PM} = NF_{CM}) \approx 30\%$.

These results didn't agree with the theoretical managerial implications.

It was late so students were asked to bright their doubts again the next time.

3. RESULTS

The model of the single unit was checked and a simulation process of 10^6 iterations was performed.

In order to verify the simulation process and the data input integrity, a comparison between theoretical and simulated results was calculated as regards to the following variables: mean value and standard deviation of random numbers which were generated in order to perform the Monte Carlo simulation process; mean value and standard deviation of times to failure (T_i, T_i').

A good agreement among theoretical and simulated results was found.

Figure 3, 4 and 5 depict further results which were obtained.

The simulation process enabled to verify the substantial identity between the distribution of number of failures f(NF_{PM}) and f(NF_{CM}*)* and how they fit very well the Poisson distribution (see figure 3); this behavior agrees to the Law of small number by which the number of rare event, along a fixed horizon, follows the Poisson distribution of the average expected number of events (in the focused case study the expected number of failures is H/MTTF=4,0) (Crathorne 1928).

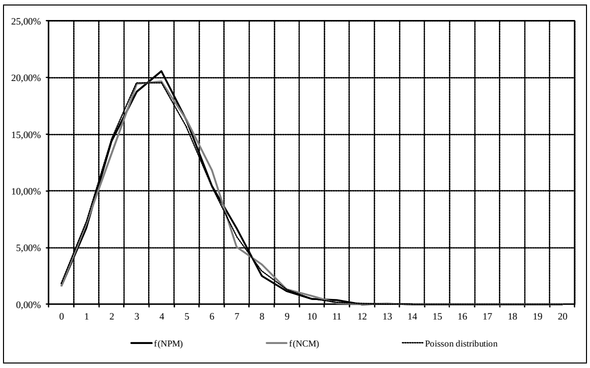

Figure 3: Failures distribution per maintenance policy

On the contrary, the difference between NF_{PM} and NF_{CM} follows an asymmetric distribution; furthermore the comparison between number of failures which are registered step by step of simulation process shows that the preventive maintenance scenarios has a more frequently number of failures which is lower than the one is registered by simulating the corrective one (see figures 4 and 5):

$F(NF_{PM}-NF_{CM} <=0) =70\%$;

$F(NF_{PM}-NF_{CM} >0) =30\%$.

Figure 4 reveals also a pseudo-Skellam behaviour; the Skellam distribution is the discrete probability distribution of the difference NF_{PM} - NF_{CM} of two statistically independent random variables NF_{PM} and NF_{CM} each having Poisson distributions with the same expected values (Skellam 1946).

The distribution is also applicable to a special case of the difference of dependent Poisson random variables, when the two variables have a common additive random contribution which is cancelled by the differencing (Karlis and Ntzoufras 2006).

The simulation process results are counter revolutionary: because they don't agree with the consolidate literature knowledge (the preventive maintenance appears to be preferred with respect to the corrective one) and because the difference between two Poisson distribution doesn't follow a Skellam function. This issue requires a deeper discussion in order to be solved. The simulation process can register data which can solve the rising issue.

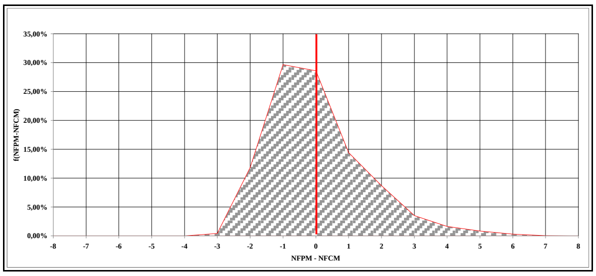

Figure 4: NF_{PM} - NF_{CM} simulated frequency distribution

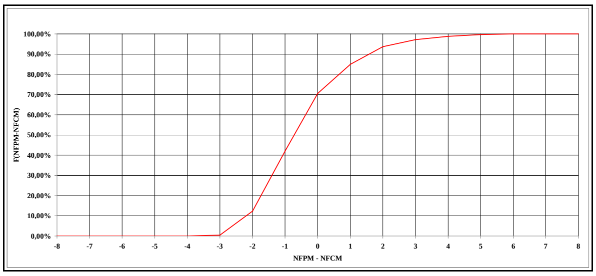

Figure 5: NF_{PM} - NF_{CM} cumulative frequency distribution

4. DISCUSSION

Figure 3 shows that the distribution frequency of failures, for both policies, seems to follow a Poisson distribution; the shape of this function is asymmetric: the probability of a number of failures which is higher than the expected one is F(NF>H/MTTF)=0,3712.

The Law of small number is respected and at the same time one can argue that the overall probability of T_i > MTTF is higher than the vice versa.

On the other hands, a high number of failures can happen with a lower overall probability, but they happen.

As regard to the difference between NF_{PM} and NF_{CM} it is to be noted that the time to failure which are random sampled for the above mentioned maintenance scenarios, Ti and Ti', are not independent:

$T_i' = T_i$ if $T_i < T_{PM}$, otherwise $T_i' = T_{PM}$

Tables 1 and 2 allow showing the dependency of the two set of variables.

A new simulation process was performed and the two maintenance models were provided with two different set of random numbers.

Figure 6 shows that when T_i and T_i' are independent variables, because they are sampled from different set of random numbers, the difference NF_{CM} - NF_{PM} follows a Skellam function.

This behaviour can be assumed as a validation of the simulation model which confirms the theoretical results when the theoretical condition of independency of input variables is established.

Although we found the reason of the pseudo-Skellam behaviour, the sequence by which items are procured and replaced in the single unit system is unique and the dependency between T_i and T_i' can not be overtaken; we can only register that a Skellam distribution doesn't occur.

In order to find the solution of the problem from a more holistic point of view was appointed.

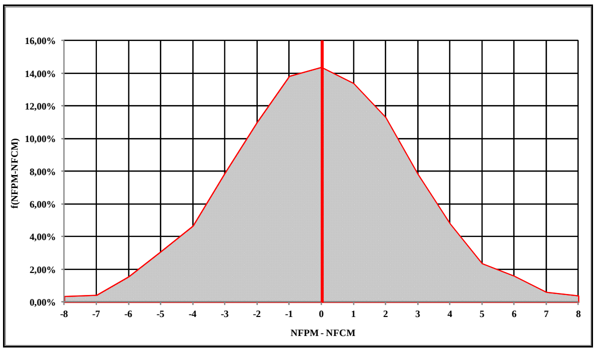

Figure 6: the Skellam NF_{CM}-NF_{PM} distribution.

Maintenance management should be defined as a risk management task: a maintenance policy is to be selected when it minimizes the risk of failures and not only the frequency of failures which belongs to a certain maintenance policy.

Let's define the risk of a maintenance policy, RoM, as:

$$RoM = f \cdot I$$

where f is the frequency according to which the scenario happens and I is the impact that can be calculated by counting the number of failures.

A final simulation process was performed; this time the model enables to register, at each step of Monte Carlo simulation, the number of failures for each kind of maintenance policy.

As regard to the preventive policy scenario, it's now possible to compute the average number of failure $NF_{PM}(NF_{PM}<NF_{CM})$ exclusively for those step of Monte Carlo simulation to which the number of failure NF_{PM} is lower than the number of failure NF_{CM}; the same calculation is performed for the opposite condition.

The comparison of the risks of maintenance policy was evaluated as follows:

$$RoM_{PM} = E(NF_{PM}(NF_{PM} < NF_{CM}))\ f(NF_{PM} < NF_{CM})$$
$$RoM_{PM}' = E(NF_{PM}(NF_{PM} > NF_{CM}))\ f(NF_{PM} > NF_{CM})$$

Figure 7 shows the evolution of the simulated risk of maintenance, RoM_{PM} and RoM_{PM}', and allows finding again that preventive and corrective maintenance policies have the same risk of maintenance when items fail according to a constant failure rate.

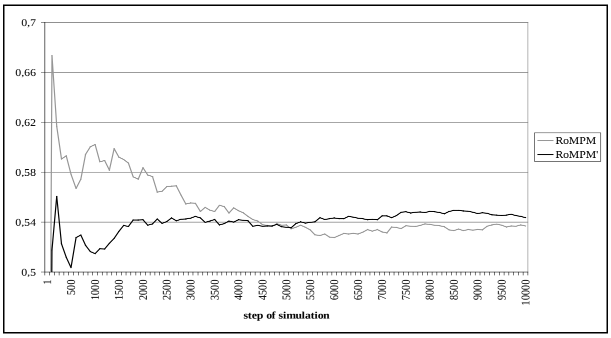

Figure7: progressive risk of maintenance RoM and RoM' (10^4 replications)

The simulation process allows discovering stochastic behaviours sometimes hidden in to the system's dynamics.

Further information could be pointed out from another high point of view: the problem was only failures dependent due the particularly relation between preventive replacement costs and failure costs; when preventive maintenance costs can be neglected if compare with the failures one, item redundancy must be considered.

When I came back to the student I was able with the same case study and the same model to make the previous doubts a new little knowledge.

CONCLUSIONS

The simulation model of maintenance policies, which was applied to a conceptual case study, allowed learning a comprehensive lesson about the behavior of the entire system.

The simulation model enabled to change point of view focusing before on the details of modeling and after on the general meaning of the process: the strident initial inconsistency of results, which appears considering from a too close point of view the problem, is overtaken trough a more general approach. This order of event seems to better lead students to learn the lesson; we would call it learning by simulating.

The nature of model and the software environment which enables to see what is get during each step of coding (Microsoft Excel®) help to capture attention from students and increases the learning empathy.

The original aim of the exercise was to point out the equivalence between the corrective and preventive maintenance of constant failure rate items.

The attempt to model and simulate the reliability system allowed pointing out some further information:

1. corrective and preventive maintenance follow equivalent risk of failure when those policies are applied to constant failure items;
2. borderline conditions, as in the case study presented which shows great failure costs, need a system assessment and not only a maintenance policy decision making;
3. the memory less property of items is rather a state of information missing than an antecedent of an elegant reliability calculation;
4. differencing two dependent Poisson distributed variables led to an asymmetric Skellam function with expected value equal to de difference of the expected values of the dependent functions.

A further analysis is requested in order to estimate which kind of failure distributions, for example the constant probability distribution, meet the small number law as the exponential one.

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