MIXING FORMALISMS BASED ON PETRI NETS FOR IMPROVING THE EFFICIENCY OF MODELLING AND SIMULATION OF DES

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ABSTRACT
The use of formal languages for describing discrete event systems provides with high quality tools for validation, verification, simulation, or optimization. Choosing and using an appropriate formalism for modeling a system is crucial for these purposes, since this choice might influence the easiness of modeling or the speed of simulation. For example, the performance of simulation-based optimization is usually very sensitive to the size of the model of the system. Large models require more computational effort and they are very common in realistic modeling and simulation.
In large models, it might be recommendable the use of different formalisms for every subsystem. This paper deals with the theoretical analysis of this situation, focussed on a paradigm, such as the Petri nets, which offer a wide range of formalisms, and an application field, such as the discrete event systems with alternative structural configurations, which usually provide with large models for developing decision support systems.

Keywords: workstation design, work measurement, ergonomics, decision support system

1. INTRODUCTION
The use of formal languages for describing discrete event systems provides the researchers and practitioners with high quality tools for relevant tasks, such as validation, verification, simulation, or optimization (Bruzzone and Longo, 2010), (Xiao and Ming, 2011), (Longo et al. 2013).

Choosing and using an appropriate formalism for developing the model of the system is a crucial stage in the application of strategies of structural analysis or performance evaluation (Balbo and Silva 1998). According to the characteristics of the system itself, it might be adequate to select a certain formal language, for improving the certain operations, such as the modeling stage or the simulation of the evolution of the system (Piera et al. 2004), (Mújica et al. 2010).

Simulation-based optimization is a usual example of operation that may require high computational effort, when the modeled system is made realistic by means of a large size of the model (Zaitsev and Shmeleva, 2011), (Latorre et al. 2013a, 2014b, 2013b) (Macias and Parte, 2004). The development of large models is usually coped by means of a modeling strategy, such as top-down or bottom-up. The resulting subsystems in which the final model can usually be divided may present very different characteristics (Silva 1993).

An efficient choice of the appropriate formalism to describe the model of a large discrete event system might require a multiple choice in the sense that diverse subsystems should be modeled using different formal languages (Latorre et al 2014a)(Jimenez et al. 2014).

This paper deals with the theoretical analysis of this situation, focused on a paradigm, such as the Petri nets, which offer a wide range of formalisms, and an application field, such as the discrete event systems with alternative structural configurations, which usually provide with large models (Silva 1993), (Recalde et al 2004), (David and Alla 2005), (Jensen and Kristensen 2009) (Jimenez et al, 2005, 2006, 2009).

The present paper is organized as follows. First of all, the concept of exclusive entity and monotypic set of exclusive entities are introduced. In section 3, a series of definitions of diverse formalisms, based on the Petri net paradigm, able to represent a set of exclusive entities is presented. In section 4 several propositions prove the ability of the mentioned formalisms for representing a set of exclusive entities. The following section deals with a key concept in this paper, the polytypic set of exclusive entities, as a formal way to introduce a variety of formalisms for describing different parts of a Petri net model. Moreover, sections 6, 7, and 8, show three case studies, where models representing monotypic, as well as polytypic sets of exclusive entities are presented. Finally, a conclusion section summarizes the achievements of the research line described in this paper, and outlines open research lines.

2. EXCLUSIVE ENTITIES
Several formalisms, based on the Petri net paradigm, have been developed for modelling discrete event systems with alternative structural configurations, such as the alternatives aggregation Petri nets, disjunctive Petri nets, compound Petri nets, or a perhaps more

intuitive set of alternative Petri nets (Latorre et al. 2013c), (Latorre et al. 2012). All of these models share in common to have an associated set of exclusive entities, representing the exclusive nature of the different structural alternative configurations (Latorre et al. 2013d).

Different transformation algorithms have been developed for converting the model of a discrete event system from one of these formalisms to another one, being both of the models equivalent, since the graphs of reachable states are the same or at least isomorphous.

This property and knowledge open the door to deal with the so called polytypic set of exclusive entities, associated to a model of the system containing subsystems expressed using different formal languages (Latorre et al. 2014c). In fact, it is possible to combine in the same model all the four formalisms mentioned in the previous paragraphs.

For example, it is possible to profit from the specific abilities of the diverse formalisms to ease the modelling of certain type of systems to deal with a bottom-up modelling process by using diverse formalisms for different subsystems. If it is required, once the modelling process has been finished, it is possible to convert, in an automatic fashion, the formalisms describing the subsystems present in the model into a single one (Latorre et al. 2014c).

More in detail, the general concept of exclusiveness, based in the idea of mutually exclusive evolution, is defined in a formal and general way by means of the concept of set of exclusive entities. Different forms of these exclusive entities allow defining different representations of disjunctive constraints equivalent to a set of alternative Petri nets; always preserving the exclusiveness between the different elements or entities.

Definition 1. Monotypic set of exclusive entities.
Given a discrete event system D, a monotypic set of exclusive entities associated to D is a set $S_x = \{ X_1, \ldots, X_n \}$, which verifies that

i) The elements of S_x are exclusive, that is to say, only one of them can be chosen as a consequence of a decision.

ii) $\forall\ i, j \in \mathbb{N}^*$, $i \neq j$ and $1 \leq i, j \leq n$ it is verified that $X_i \neq X_j$.

iii) The elements of S_x are of the same type.

iv) $\exists$ f: $S_x \rightarrow S_R$ such that

iv.a) $S_R = \{ R_1, \ldots, R_n \}$ is a set of alternative Petri nets, feasible models of D.

iv.b) f is a bijection $\Rightarrow \forall\ X_i \in S_x\ \exists!\ f(X_i) = R_i \in S_R$ such that R_i is a feasible model for D and $\forall\ R_i \in S_R\ \exists!\ f^{-1}(R_i) = X_i \in S_x$.

□

The purpose of a set of exclusive entities is to represent in different ways the exclusive Petri net models of a discrete event system. A supposition is made on these alternative models: they have a different static structure. For this reason, as it happens in the real practice, if the difference is, for example, the initial state, the models are supposed to be the same.

The exclusive entities may be understood as an abstraction of the feasible formal representations for the alternative Petri nets that can be chosen for a DES. This doctoral thesis is devoted to find different forms of the exclusive entities, their properties, transformation algorithms between them, their feasible combinations and their performance in the solution of optimization problems.

3. PETRI NET FORMALISMS ABLE TO REPRESENT EXCLUSIVE ENTITIES

Several formalisms, based on the paradigm of the Petri nets, have been defined and used for the description of discrete event systems with alternative exclusive entities.

The most natural and intuitive way to represent a discrete event system with freedom degrees related to alternative structural configurations is a set of alternative Petri nets. According to this approach, it is possible to obtain a different model for the system associated to every one of the structural configurations.

An important property verified by the elements of a set of alternative Petri nets is the following.

Definition 2. Mutually exclusive evolution.
Given two different Petri nets R and R'. They are said to have mutually exclusive evolutions if it is verified:

i) If $\mathbf{m}(R) \neq \mathbf{m_0}(R) \Rightarrow \mathbf{m}(R') = \mathbf{m_0}(R')$

ii) If $\mathbf{m}(R') \neq \mathbf{m_0}(R') \Rightarrow \mathbf{m}(R) = \mathbf{m_0}(R)$

□

Based on the previous definition, it is possible to define a set of alternative Petri nets in the following way:

Definition 3. Set of alternative Petri nets.
Given a set of Petri nets $S_R = \{ R_1, \ldots, R_n \}$, S_R is said to be a set of alternative Petri nets if $n>1$ and $\forall\ i, j \in \mathbb{N}^*$ such that $i \neq j$, $1 \leq i, j \leq n$, it is verified that

i) R_i and R_j have mutually exclusive evolution.

ii) $\mathbf{W}(R) \neq \mathbf{W}(R')$. That is to say, the structure of the Petri nets, represented by their incidence matrices, is different.

R_i is called the *i*-th alternative Petri net of S_R.

□

Another formalism, able to represent a set of exclusive entities, is the compound Petri net. It can be seen as a parametric Petri net, where some of the parameters belong to the structure of the net and not to its behaviour.

Definition 4. Compound Petri net.

A compound Petri net is a 7-tuple

$R^c = \langle P, T, W^+, W^-, \mathbf{m_0}, S_\alpha, S_{val\alpha} \rangle$, where

i) P and T are disjoint, finite, non-empty sets of places and transitions respectively.

ii) $W^-: P \times T \rightarrow \mathbb{N}$ is the pre-incidence or input function.

iii) $W^+: T \times P \rightarrow \mathbb{N}$ is the post-incidence or output function.

iv) $\boldsymbol{m}_0$ is the initial marking of the net that represents the initial vector of state and is usually a function of the choice variables.

v) S_α is the set of undefined parameters of R^c.

vi) $S_{str\alpha} \neq \emptyset$ is the set of undefined structural parameters of R^c, such that $S_{str\alpha} \subseteq S_\alpha$.

vii) $S_{val\alpha}$ is the feasible combination of values for the undefined parameters .

□

Furthermore, a formalism, able to represent sets of exclusive entities, and appropriate to model discrete event systems, which verify that the alternative Petri nets share complete subnets is the alternatives aggregation Petri net.

Definition 5. Alternatives aggregation Petri net system.

An alternatives aggregation Petri net system, R^A, is defined as the 10-tuple:

$$R^A = \langle P, T, W^+, W^-, \mathbf{m}_0, S_A, \mathrm{f}_\mathrm{A}, R_{val\gamma} \rangle$$

Where,

The first four elements in the definition are the same of the ones seen in the previous definition of compound Petri net.

i) $\boldsymbol{m}_0$ is the initial marking of the net and is usually a function of the choice variables.

ii) S_A is a set of choice variables such that $S_A \neq \emptyset$ and $|S_A| = n$.

iii) f_A: $T \rightarrow \mathrm{f}(a_1, \ldots, a_n)$ is a function that assigns a function of the choice variables to each transition t such that type[$\mathrm{f}_\mathrm{A}(t)$] = boolean.

iv)$R_{val\gamma}$ is a binary relation between $S_{val\gamma}$ and R_A.

□

4. FORMS OF A SET OF EXCLUSIVE ENTITIES

The set of exclusive entities S_x can take different forms. On the other hand, all the elements of S_x must be of the same nature or structure: Petri nets, binary or Boolean variables, colours of tokens or integer numbers.

In this section, several propositions will be given, for justifying the fact that the formalisms based on the Petri net paradigm, enumerated in the previous section can, in fact, represent a set of exclusive entities.

Proposition 1. A set of alternative Petri nets as a monotypic set of exclusive entities.

Let D be a discrete event system.

Let S_R be a set of alternative Petri nets such that $\forall\ R_i \in S_R$, R_i is a feasible model of D.

It is verified that S_R is a set of exclusive entities associated to D.

Proof

Being S_R a set of alternative Petri nets, models of D with exclusive evolution, they are alternative models for D. This statement means that in order to define univocally the model of D it is necessary to make a decision on a $R_i \in S_R$. As a consequence *i*) and *iii*.a) are verified.

The elements of S_R are Petri nets hence they belong to the same type. Thus, *ii*) is verified.

□

Proposition 2. A set of choice variables as a monotypic set of exclusive entities.

Let S_R be a set of alternative Petri nets, feasible models of a discrete event system D.

Let S_A be a set of choice variables.

If $|S_A| = |S_R| \Rightarrow S_A$ is a monotypic set of exclusive entities.

Proof

By definition, the elements of the set S_A are exclusive, since only one of them can be set to 1 as a consequence of a decision. Then *i*) is verified.

The elements of S_A are binary or Boolean variables hence they belong to the same type.

As $|S_A| = |S_R|$ is possible to define a bijection f: $S_A \rightarrow S_R$ such that

$\forall\ a_i \in S_A\ \exists!\ \mathrm{f}(a_i) = R_i \in S_R$ such that R_i is a feasible model for D

$\forall\ R_i \in S_R\ \exists!\ \mathrm{f}^{-1}(R_i) = a_i \in S_A$.

□

Proposition 3. A set of feasible combinations of values for a set of undefined structural parameters as a monotypic set of exclusive entities.

Let R^c be a compound Petri net, developed as model of a DES.

Let $S_{valstr\alpha}$ be a set of undefined structural parameters of a compound Petri net.

Let S_R be a set of alternative Petri nets, associated as models of a discrete event system D, that arise when the undefined structural parameters of R^c take values from the different feasible combinations of values belonging to $S_{valstr\alpha}$.

$S_{valstr\alpha}$ is a monotypic set of exclusive entities.

Proof

By definition, the elements of the set $S_{valstr\alpha}$ are exclusive, since only one of them can be chosen at a time as a consequence of a decision.

The elements of $S_{valstr\alpha}$ re integers. As they belong to the same type, *ii*) is verified.

By construction of S_R $|S_{valstr\alpha}| = |S_R|$, then it is possible to define a bijection f: $S_{valstr\alpha} \rightarrow S_R$ such that
$\forall\ cv_i \in S_A\ \exists!\ f(cv_i) = R_i \in S_R$ such that R_i is a feasible model for D
$\forall\ R_i \in S_R\ \exists!\ f^{-1}(R_i) = cv_i \in S_{valstr\alpha}$.

□

Notice that in this last proposition a set of alternative Petri nets has been built up from a compound Petri net. The proof may be performed as well by the construction of the compound Petri net from the merging of a set of matching alternative Petri nets from an original set of alternative Petri nets.

5. POLYTIPIC SET OF EXCLUSIVE ENTITIES

Definition 4. Polytypic set of exclusive entities.
Given a discrete event system, a polytypic set of exclusive entities associated to it is a set $S_x^p = \{ X_1, \ldots, X_n \}$, which verifies that

i) The elements of S_x are exclusive, that is to say, only one of them can be chosen as a consequence of a decision.

ii) $\exists\ S_x$, $S'_x \subseteq S_x^p$ such that the elements in S_x and in S'_x are of different types.

iii) $\exists$ f: $S_x^p \rightarrow S_R$ such that

iii.a) $S_R = \{ R_1, \ldots, R_n \}$ is a set of alternative Petri nets, feasible models of D.

iii.b) f is a bijection $\Rightarrow \forall\ X_i \in S_x^p\ \exists!\ f(X_i) = R_i \in S_R$ such that R_i is a feasible model for D and $\forall\ R_i \in S_R\ \exists!\ f^{-1}(R_i) = X_i \in S_x^p$.

□

Note that the only difference between "Definition 1. Monotypic set of exclusive entities" and "Definition 4. Polytypic set of exclusive entities" is the condition *ii)*. In other words, the monotypic set includes exclusive entities of a single type, for example Petri nets, choice variables or feasible combinations of values for the undefined structural parameters. On the other hand, a polytypic set of exclusive parameters contains elements of different types, for example Petri nets and choice variables.

In order to illustrate the concept of polytypic set of exclusive entities and its potential for easing the modeling of large and complex discrete event systems, in the following sections, a set of case studies will be presented.

6. CASE STUDY 1

As first example in the illustration of the concepts presented in this paper, it will be developed a set of alternative Petri nets.

Let us consider the design of a certain discrete event system, where, after a first stage in the design process, a number of twelve alternative structural configurations have been identified.

A first, natural, and intuitive way to proceed for obtaining a model of this system consists in the development of a set of alternative Petri nets. Every structural configuration will have a counterpart as an alternative Petri net.

However, this approach for the modeling process of a discrete event system with alternative structural configurations may be too intensive in time consumption, since a model of the system should be developed for every feasible solution in the pool of alternative structural configurations.

The set of exclusive entities in this model is a monotypic set of exclusive entities, since all the exclusive entities are alternative Petri nets; hence, they are of the same type.

In figures 1 to 3 some of the twelve alternative Petri nets are shown, only in their graphical representation. However, obtaining from this figures the matrix-based representation is trivial.

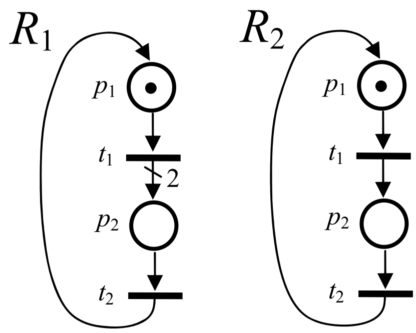

Figure 1: Alternative Petri nets R_1 and R_2

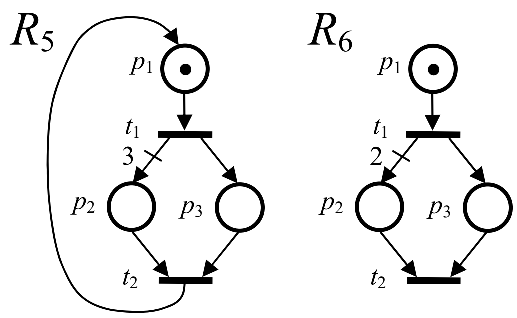

Figure 2: Alternative Petri nets R_5 and R_6

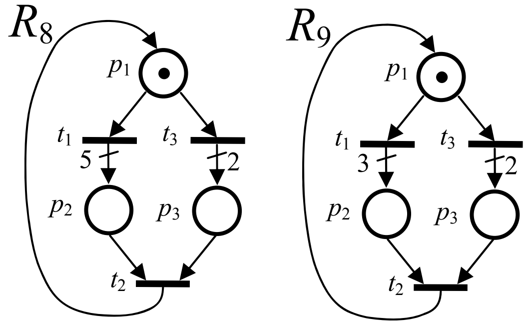

Figure 3: Alternative Petri net R_8 and R_9

7. CASE STUDY 2

In this section, the formalism of the compound Petri nets will be applied for profiting from its advantages for transforming the model of the discrete event system developed in the previous section into another one more compact and better for performing simulation-based operations, such as performance analysis or optimization.

Furthermore, for the modeling process illustrated in the previous section, the compound Petri nets might have been used in conjunction with the alternative Petri nets. Combining both formalisms, the modeling process is also simplified, since it is not necessary to develop different models for every alternative structural configuration. In fact, compound Petri nets allow developing a single Petri net model for a set of alternative structural configurations.

In figures 4 to 6, the three compound alternative Petri nets of the model can be found: $S_R^c = \{ R_1^c , R_2^c , R_1^c \}$.

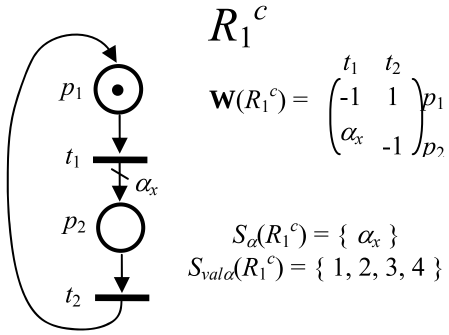

Figure 4: Alternative Petri net R_1^c

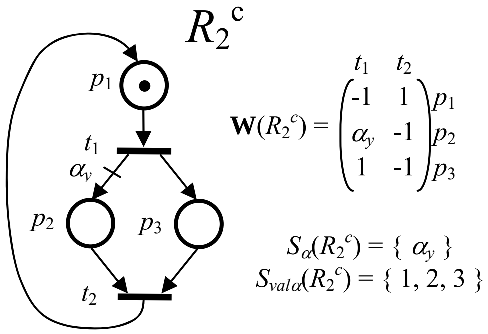

Figure 5: Alternative Petri net R_2^c

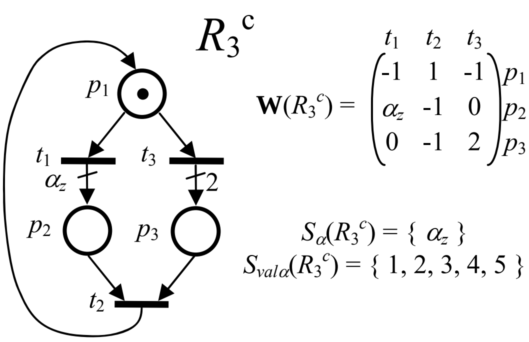

Figure 6: Alternative Petri net R_3^c

8. CASE STUDY 3

The model presented in the previous section is a set of alternative compound Petri nets. In that model, some of the exclusive entities are represented by means of three alternative Petri nets, while the rest of the exclusive entities, totalizing a number of 12, are represented by sets of feasible values for the undefined structural parameters of the compound Petri nets: 4, 5, and 3, respectively. As it has been proven, the model has an associated polytypic set of exclusive entities.

In this section, the three alternative compound Petri nets will be reduced into a single compound Petri net, including undefined structural parameters. In this case, the exclusive entities are not represented by alternative Petri nets anymore but the twelve of them by feasible combinations of values for the undefined structural parameters. Again, as it happened in section 6, the model of the original discrete event system, has included a monotypic set of exclusive entities.

The purpose of this transformation of the model, from a set of alternative compound Petri nets to a single compound Petri net may be, for example, to assess the improvement (or not) in the computer time required to perform a simulation for developing a performance evaluation.

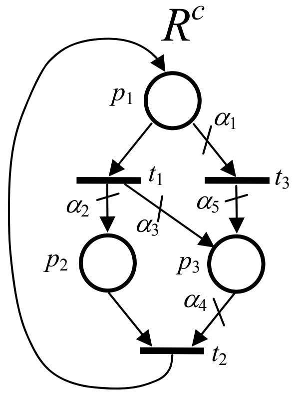

Figure 7: Graphical representation of the compound Petri net R^c

$$\mathbf{W}(R^c) = \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{pmatrix} -1 & 1 & -\alpha_1 \\ \alpha_2 & -1 & 0 \\ \alpha_3 & -\alpha_4 & \alpha_5 \end{pmatrix} \end{array} \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array}$$

Figure 8:Matrix-based representation of the compound Petri net R^c

The set of undefined parameters of the compound Petri net contains five structural parameters $S_\alpha(R^c) = S_{str\alpha}(R^c) = \{ \alpha_1 , \alpha_2 , \alpha_3 , \alpha_4 , \alpha_5 \}$. On the other hand, the set of feasible combinations of values for the undefined structural parameters has twelve elements: $S_{valstr\alpha}(R^c) = \{$ (0,1,0,0,0) , (0,2,0,0,0) , (0,3,0,0,0) , (0,4,0,0,0) , (0,1,1,1,0) , (0,2,1,1,0) , (0,3,1,1,0) , (1,1,0,1,2) , (1,2,0,1,2) , (1,3,0,1,2) , (1,4,0,1,2) , (1,5,0,1,2) }.

In figure 7, it is possible to find the graphical representation of the resulting compound alternative Petri net, while in figure 8, its matrix-based representation is shown.

9. CONCLUSIONS

This paper has presented the concept of monotypic and polytypic sets of exclusive entities, as an important consideration for improving the efficiency of simulation-based operations, such as performance evaluation or optimization, using Petri net models for describing discrete event systems with alternative structural configurations.

The previous sections have overviewed a number of formalisms, able to cope with sets of exclusive entities, and some of them have been used in three case studies for showing both monotypic and polytypic sets of exclusive entities, as well as some of the advantages of using one approach or the other one.

In particular a set of 12 alternative Petri nets have been considered in the first case study, while in a second example, these Petri nets have been merged into three compound Petri nets profiting from the structural similarities of the original nets. As a result, a set of three alternative compound Petri nets have been obtained.

In the last case study, the three alternative compound Petri nets have been merged into a single compound Petri net, transforming the previous polytypic set of exclusive entities into a monotypic one.

From a theoretical point of view, the present paper has proven that any of the formalisms presented in this paper can represent a set of exclusive entities. These results are the basis that allow the combination of some of these formalisms into a single model of a discrete event system.

As open research lines, it can be considered the analysis of other combinations of formalisms not considered in this paper, as well as their application to real problems, where the size of the model can be increased in orders of magnitude.

REFERENCES

Balbo G and Silva M (eds). 1998. Performance Models for Discrete Event Systems with Synchronizations: Formalisms and Analysis Techniques. Saragosse, Spain: Kronos, 1998.

Bruzzone A.G. and Longo F. 2010. An advanced system for supporting the decision process within large-scale retail stores. Simulation; 86: 742–762.

David R and Alla H. 2005. Discrete, Continuous and Hybrid Petri Nets. Berlin: Springer.

Jensen, K., Kristensen, L.M. 2009. Colored Petri nets. Modelling and Validation of Concurrent Systems, Springer.

Jimenez, E., Martinez, E., Blanco, J., Perez, M., & Graciano, C. (2014). Methodological approach towards sustainability by integration of environmental impact in production system models through life cycle analysis: Application to the Rioja wine sector. Simulation-Transactions of the Society for Modeling and Simulation International, 90, 143-161

Jiménez, E., Pérez, M., & Latorre, I. (2005). On deterministic modelling and simulation of manufacturing systems with petri nets. Paper presented at the International Mediterranean Modeling Multiconference, I3M 2005, European Modeling Simulation Symposium, EMSS 2005.

Jiménez, E., Pérez, M., & Latorre, I. (2006). Industrial applications of petri Nets: System modelling and simulation. Paper presented at the International Mediterranean Modelling Multiconference, I3M.

Jiménez, E., Pérez, M., & Latorre, J. I. (2009). Modelling and simulation with discrete and continuous PN: Semantics and delays. Paper presented at the 21st European Modeling and Simulation Symposium, EMSS 2009.

Latorre, J.I. and Jiménez, E. 2013. Simulation-based optimization of discrete event systems with alternative structural configurations using distributed computation and the Petri net paradigm. Simulation. 2013 89 (11), pp. 1310-1334

Latorre, J.I. and Jiménez, E., Blanco, J., Sáenz, J. C. 2013. Decision Support in the Rioja Wine Production Sector. International Journal of Food Engineering. Volume 9, Issue 3 (Jun 2013). Page 267.

Latorre, J.I. and Jiménez, E., Blanco, J., Sáenz, J. C. 2014. Optimal Design of an Olive Oil Mill by Means of the Simulation of a Petri Net Model. International Journal of Food Engineering. Published online, May 2014.

Latorre, J.I., Jiménez, E. 2012. Colored Petri Nets as a Formalism to Represent Alternative Models for a Discrete Event System. 24th European Modelling and Simulation Symposium (EMSS 12). Vienna, 2012.

Latorre, J.I., Jiménez, E., de la Parte, M., Blanco, J., Martínez, E. 2014. Control of Discrete Event Systems by Means of Discrete Optimization and Disjunctive Colored PNs: Application to Manufacturing Facilities. Abstract and Applied Analysis. Volume 2014, 16 pages.

Latorre, J.I., Jiménez, E., Pérez, M. 2013. The optimization problem based on alternatives aggregation Petri nets as models for industrial discrete event systems. Simulation. March 2013 89 (3), pp. 346-361.

Latorre, J.I., Jiménez, E., Pérez, M. 2014. Sequence of decisions on discrete event systems modeled by Petri nets with structural alternative configurations. Journal of Computational Science. 5(3): 387-394 (2014).

Latorre, J.I., Jiménez, E., Pérez, M., “Petri nets with exclusive entities for decision making”.

International Journal of Simulation and Process Modeling, Special Issue on Multidisciplinary Approach to Complex Systems Design and Management: Modelling and Simulation based on Methodologies and Tools. 2013 Vol. 8 No. 1

Longo, F., Nicoletti, L., Chiurco, A., Solis, A. O., Massei, M., Diaz, R. 2013. Investigating the behavior of a shop order manufacturing system by using simulation. SpringSim (EAIA) 2013: 7

Macias, E. J., & de la Parte, M. P. (2004). Simulation and optimization of logistic and production systems using discrete and continuous Petri nets. Simulation-Transactions of the Society for Modeling and Simulation International, 80, 143-152.

Mújica M. A., Piera M.A., and Narciso M. 2010. Revisiting state space exploration of timed coloured Petri net models to optimize manufacturing system's performance. Simulation Modelling Practice Theory 2010; 18: 1225–1241.

Piera, M.À., Narciso, M., Guasch, A., and Riera, D. 2004. Optimization of logistic and manufacturing system through simulation: A colored Petri net-based methodology. Simulation, vol. 80, number 3, pp 121-129, May 2004

Recalde, L.; Silva, M.; Ezpeleta, J.; and Teruel, E. 2004. Petri Nets and Manufacturing Systems: An Examples-Driven Tour. In: Lectures on Concurrency and Petri Nets: Advances in Petri Nets, pages 742-788. Volume 3098 of Lecture Notes in Computer Science / Desel, J.; Reisig, W.; and Rozenberg, G. (editors). Springer-Verlag, June 2004.

Silva, M. "Introducing Petri nets", In Practice of Petri Nets in Manufacturing", Di Cesare, F., (editor), pp. 1-62. Ed. Chapman&Hall. 1993.

Xiao, Z. and Ming, Z. 2011. A method of workflow scheduling based on colored Petri nets. Data & Knowledge Engineering 70, pp. 230–247. Elsevier. 2011.

Zaitsev, D.A., Shmeleva, T.R. 2011. A Parametric Colored Petri Net Model of a Switched Network. Int. Journal of Communications, Network and System Sciences, 4, 65-76, Scientific Research Publishing Inc., 2011.