STOCHASTIC ODE AND PDE MODELS OF THE CURRENT STOCK OF DIVISIBLE PRODUCTIONS

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ABSTRACT
In the given paper we investigate the problem of constructing continuous and unsteady mathematical models for determine the volumes of current stock of divisible productions in one or several interconnected warehouses using apparatus of mathematical physics and continuum principle. It is assumed that time of production distribution and replenishment is continuous. The constructed models are stochastic, and have different levels of complexity, adequacy and application potentials. The simple model is constructed using the theory of ODE, for construction of more complex models it is applied the theory of PDE. Besides, provided some additional conditions the finite-differenced model for determination of random volume of divisible homogeneous production is constructed, and this finite differenced mathematical model makes possible to determine one of possible trajectories of the random quantity. All constructed models can be used for on-line monitoring of the dynamics of the productions random volumes.

Keywords: inventory control model, current stock, divisible production, equations of mathematical physics

1. INTRODUCTION

One of the central problems of the inventory control theory is to find an optimal or quasi optimal solution to the task of ordering productions to be supplied, and main result of the task is the answer on two basic questions: how much to order and when to order. Of no less interest it is the task of determining the current stock of certain production (sold by the piece or indivisible production and dry or divisible production) at any given moment of a fixed time span, with any random factors taken into account. By "current stock" we denote the quantity (volume) of the production accumulated in the stock, which is used for regular distribution (i.e. replenishment). Quite a lot of different types of models of varying complexity, purpose and adequacy have been developed in the inventory control theory. Most of the existing mathematical models in this theory consider indivisible prodution (Kopytov et al 2007; Ashmanov 1980; Nikaido 1968). We can classify these models taking in account different their properties: deterministic and stochastic, linear and nonlinear, single- and multi-product, discrete and continuous models, and etc. (Nikaido 1968).

The present paper studies construction of continuous and unsteady mathematical models for calculating the volume of current stock of divisible production "from scratch" using apparatus and equations of mathematical physics. The suggested models are stochastic ones and have different levels of complexity, adequacy and application potentials. The simple models are constructed using the theory of ordinary differential equations, for construction of more complex models it is applied the theory of partial differential equations.

2. STOCHASTIC ODE MODEL FOR DETERMINING THE VOLUME OF THE CURRENT STOCK OF THE HOMOGENEOUS DIVISIBLE PRODUCTION

In the present section we construct the continuous stochastic mathematical model for determining the volume of current stock of divisible production. For this purpose, we will use the apparatus of mathematical physics and the continuum principle (Tikhonov and Samarsky 2004); as modelling language will be chosen language of ODE. Before introducing the simplifying assumptions, which are required for modeling, as well as variables, parameters and functions that describing and coupling the initial data of the simulated process with unknown quantities of the current stock dynamics, we will consider briefly the issue of stochasticity of the mathematical model under construction. Namely, to construct the stochastic (i.e. not deterministic) model, we can proceed in the following two ways:

– the current stock to be determined is not supposed to be an accidental quantity, but after the introduction of a change rate the constructed model is supplied with all random factors which visibly influence unknown rate of the current stock change. In this case the obtained relation (in the form of the above mentioned ODE) with regard to unknown volume of the current stock and rate of its change is a functional relationship among unknown volume, rate of its change, and accidental quantities (factors) influencing the current stock dynamics. In other words, in the obtained model, unknown volume, which initially did not seem to be assumed as an accidental value (stochastic value of a random function, to be more specific), due to the obtained ODE and corresponding conditions (initial conditions, conditions of co-ordination, etc.) appears dependent on the random quantities taken into account, i.e. unknown volume of the current stock is a function of the accidental quantities;

– the current stock is initially taken to be a random quantity, and this suggestion is taken into account when constructing the model.

The first of these ways is selected for the description of the mathematical model that will follow. It is worth mentioning in the way of a preliminary note that this choice will result in the construction of a stochastic model represented by the Ito-type differential equation (Ito 1987; Milstein 1995; Kuznetsov 2007; Diend 1990).

Now we can start constructing the mathematical model "from the scratch". Let us assume that the current stock volume of the considered homogeneous divisible production at the moment t equals to $x(t)$. It is required that $x(t)\in C[T_s,T_e];\ \exists x'(t)\ \forall t\in[T_s,T_e]$ where $[T_s,T_e]$ is a segment of time during which the dynamics of the current stock change is being studied, by T_s and T_e we denote the initial and final moments of this period of time, respectively. The requirement $x(t)\in C[T_s,T_e]$ is easy to interpret economically, and it is met if we assume that the current stock $x(t)$ is being constantly distributed/replenished. The requirement $\exists x'(t)\ \forall t\in[T_s,T_e]$ is a purely mathematical one, i.e. it is necessary to ensure a mathematical correctness of the model.

If an increase of the current stock volume $x(t)$ is as

$$\Delta x(t)\overset{def}{\equiv} x(t+\Delta t)-x(t),\ \Delta t>0,\ t+\Delta t\le T_e,$$

then

$$\frac{dx(t)}{dt}\equiv\lim_{\Delta t\to 0}\frac{\Delta x(t)}{\Delta t}, \tag{1}$$

and this quantity designates the change rate of the current stock volume at a given time t.

The rate $\frac{dx(t)}{dt}$ derived from (1) is completely analogous to the rate of a material point of continuous medium moving in metric space. It is then useful to find out the factors or reasons causing the change $x(t)$ and, consequently, trigger the existence of $\frac{dx(t)}{dt}$.

With this aim in view, the following functions are introduced: $S(t,x(t))$ describing a continuous replenishment of the current stock and $C(t,x(t))$ describing a continuous distribution of the current stock. Then the difference $S(t,x(t))-C(t,x(t))$ is a measure of the change of the current stock volume, i.e.

$$\frac{dx(t)}{dt}=S(t,x(t))-C(t,x(t)). \tag{2}$$

Let us work out the functions that make up the right side of the equation (2), namely functions $S(t,x(t))$ and $C(t,x(t))$ in detail. The function of continuous replenishment $S(t,x(t))$ consists of three additive components, namely, from regulated replenishment of the stock, which is designated as $S_{reg.}(\bullet)$; from unregulated replenishment $S_{unreg.}(\bullet)$; and from random replenishment (for instance, a random stock replenishment due to an exceptionally high quality of production or because of an expected sudden deficit of particular products, etc.), which can be described mathematically as a random quantity $X_S(t)$ that designating the total volume of production that have been delivered into a particular warehouse from random and/or non-random sources by the time t , with all random circumstances taken into account. It is assumed for all types of replenishment that all orders are instantaneously executed, i.e. the shipping time for particular supplies is not considered in the present work. Let us interpret the introduced functions:

1) the function $S_{reg.}(\bullet)$ can be interpreted as "one hundred per cent" (guaranteed) constant replenishment of the current stock of divisible production, i.e. replenishment of the current stock that takes place regularly according to a contract during the segment $[T_s,T_e]$, with the volume of such replenishment being either constant (i.e. $S_{reg.}\equiv const.$) or depending on t (i.e. being a function of the argument time $S_{reg.}=S_{reg.}(t)$, or else being functionally dependent on $x(t)$ (i.e. $S_{reg.}=S_{reg.}(t,x(t))$);

2) the function $S_{unreg.}(\bullet)$ obviously depends on t and functionally on $x(t)$, and also on a certain quantity $x_0(t)$, which designates the minimal volume of stock in a particular warehouse necessary for administering unregulated stock replenishment on condition that such replenishment is guaranteed. In other words, $S_{unreg.}=S_{unreg.}(t,x(t),x_0(t))=k_0\cdot x(t)\cdot\delta(x(t),x_0(t))$, where k_0 is a proportion coefficient, and the function $\delta(x(t),x_0(t))$ is an indicator function, which has the form

$$\delta(x(t),x_0(t))=\begin{cases}1, & if\ x(t)\le x_0(t),\\ 0, & if\ x(t)>x_0(t);\end{cases} \tag{3}$$

3) the random quantity $X_S(t)$ determines the total volume of production that was delivered into the

warehouse by the time t due to random circumstances from random and/or non-random sources. Then the quantity $X_S(t+\Delta t)$ designates the sum total of all random deliveries by the time $t+dt$, where dt is an elementary interval of time (on analogy with the terminology of mathematical physics), and $0<dt\ll 1,\ t+dt\le T_e$. Consequently, it is possible to introduce a stochastic differential of a random process $X_S(t)$, namely, the quantity

$$dX_S dt \overset{def}{\equiv} X_S(t+dt)-X_S(t),$$

which determines a random addition to the current stock of divisible productions during the elementary interval of time dt.

Now the function $C(t,x(t))$ that is contained in the right-hand side of the equation (2) and describes the dynamics of the continuous distribution of the current stock of divisible productions can be looked at in more detail. The function of continuous distribution $C(t,x(t))$ consists of four additive components: regulated distribution which is denoted as $C_{reg.}(\bullet)$; unregulated distribution $C_{unreg.}(\bullet)$; possible losses $C_{\text{loss}}(\bullet)$ of divisible productions which take place during holding and distribution processes (for example, for petroleum productions it is evaporation, for grain main reasons of losses are gnawing animals and inundation); and random distributions (similar to random replenishment, there can be circumstances due to which random distribution takes place) that can be mathematically presented as a random quantity $X_C(t)$ designating the total volume of productions that was taken away from the warehouse by the time t due to random circumstances. Let us now interpret the introduced functions:

1) the function $C_{reg.}(\bullet)$ can be interpreted as "strong" (guaranteed) constant distribution of the current stock of divisible productions, i.e. the volume of the current stock that is regularly taken away from the warehouse according to contracts during the segment $[T_s,T_e]$, with the volume of such distribution being either constant (i.e. $C_{reg.}\equiv const.$) or depending on t (i.e. being a function of the argument time $C_{reg.}=C_{reg.}(t)$, or else being functionally dependent on $x(t)$ (i.e. $C_{reg.}=C_{reg.}(t,x(t))$);

2) the function $C_{unreg.}(\bullet)$ depends on the time t and functionally on $x(t)$ in general, as well as on a certain threshold function $x_1(t)$, which determines the stock volume of divisible productions allowing for its unregulated distribution, $C_{unreg.}=C_{unreg.}(t,x(t),x_1(t))$. In order to find an analytical expression of the function $C_{unreg.}(t,x(t),x_1(t))$ the following assumptions can be made:

- under $x(t)\to\infty$ must be $C_{unreg.}(t,x(t),x_1(t))\to k_1$, where the quantity k_1 is the capacity of distributing the stock volume of divisible productions from the warehouse in the sense that whatever the stock replenishment (i.e. the quantity $\max\limits_{t\in[T_s,T_e]} S(t,x(t))$, the warehouse can not possibly distribute the stock of divisible productions measured as k_1 during the entire considered time segment $[T_s,T_e]$;
- under $x(t)\to k_2\equiv const.$, where k_2 is an averaged value of the replenishment volume that allows for unregulated distribution, must be

$$C_{unreg.}(t,x(t),x_1(t))\to\begin{cases}0, & if\quad x_1(t)\ge k_2,\\ \dfrac{k_1}{2}, & if\quad x_1(t)<k_2.\end{cases}$$

The last two suppositions allow for determining unknown analytical form of the function $C_{unreg.}(t,x(t),x_1(t))$:

$$C_{unreg.}(t,x(t),x_1(t))=k_1\cdot x(t)\cdot\frac{1-\delta(x(t),x_1(t))}{x(t)+k_2},$$

where the indicator function $\delta(x(t),x_1(t))$ has the same sense/value as in determining the function $S_{unreg.}(t,x(t),x_0(t))$; it is derive by formula (3) with the corresponding substitution of $x_1(t)$ for $x_0(t)$;

3) the function $C_{loss}(\bullet)$ describes possible losses of the divisible productions current stock in storage and distribution. For instance, if we have the oil productions stock, losses will result from the evaporation and/or from the leakage through the reservoirs; if we have the agricultural productions stock (wheat, rice, meal, and the like), there will be unavoidable losses caused by pests, flood, strong winds, etc. Apparently, the value of these losses is a random one. Though, concluding the expression for the losses' function $C_{loss}(\bullet)$, we consider it reasonable to split these losses into somehow "normal/predictable" losses and into "abnormal/extraordinary" ones. Reasonability of this splitting lies in the following: any enterprise engaged in the processes of storing/distributing of the divisible vulnerable productions (for example, perishables, productions attractive for insects and rodents and the like, productions easily affected by winds and humidity and the like) should envisage in their model the parameter describing the value of the "normal/predicted" loss (for instance, in the form of a constant – the upper limit of a possible loss; in the form of a linear function – some reasonable and somehow unpredictable percent of loss of the total productions volume; etc.). But besides this accounted/envisaged value of the "normal/predictable" loss in the processes of storing-distributing of the divisible productions there

may occur some other a priori unpredictable losses which damage may greatly exceed that of the "normal/predicted" loss. Therefore, in the given paper we'll be building the loss functions $C_{loss}(\bullet)$ with the account of both normal/predictable" losses and "abnormal/extraordinary" ones. Function $C_{loss}(\bullet)$ in the general case depends on the time t as on the argument; functionally on the volume of the current stock $x(t)$; and on two more functions $x_2(t)$ и $x_3(t)$, which meaning is the following. Function $x_2(t)$ determines the most favorable scenario of the loss at each fixed moment of time $t \in [T_s, T_e]$; this function, as said above, may be chosen, for example, as a constant – the upper limit of the possible "normal/predictable" loss, or as a linear function – a predicted loss percent of the total production volume. Function $x_3(t)$ determines a less favorable scenario (average, for example) of the loss at each fixed moment of time $t \in [T_s, T_e]$.

Thus, $C_{loss} = C_{loss}(t, x(t), x_2(t), x_3(t))$. To find the analytical expression of the function $C_{loss}(t, x(t), x_2(t), x_3(t))$, we'll make the following assumptions:

- at $x(t) \to \infty$, that is at increasing the volume of the current stock, the formula $C_{loss}(t, x(t), x_2(t), x_3(t)) \to \max\limits_{t\in[T_s,T_e]} x_2(t)$ must be valid, where $\max\limits_{t\in[T_s,T_e]} x_2(t)$ is the power of the loss for the whole considered period of time $[T_s, T_e]$;
- if $x(t) \le x_2(t),\ t \in [T_s, T_e]$, the loss process must stop, i.e. $C_{loss}(t, x(t), x_2(t), x_3(t)) \equiv 0$;
- if $x(t) > x_2(t),\ t \in [T_s, T_e]$, than under the condition $x(t) \to x_3(t),\ t \in [T_s, T_e]$ the formula $C_{loss}(t, x(t), x_2(t), x_3(t)) \to \frac{1}{2} \cdot \max\limits_{t\in[T_s,T_e]} x_2(t)$ must be valid.

The three former assumptions allow us to determine the requested analytical form of the function $C_{loss}(t, x(t), x_2(t), x_3(t))$:

$$C_{loss}(t, x(t), x_2(t), x_3(t)) = \\ = \max_{t\in[T_s,T_e]} x_2(t) \cdot x(t) \cdot \frac{1-\delta(x(t), x_2(t))}{1+\dfrac{x_3(t)-x_2(t)}{x(t)-x_2(t)}},$$

where the indicator function $\delta(x(t), x_2(t))$ is determined by formula (3) at a corresponding change of $x_0(t)$ for $x_2(t)$;

4) the random quantity $X_C(t)$ designates the total volume of productions that has been removed from the warehouse by the time t due to random circumstances. Then $X_C(t+\Delta t)$ designates the sum total of random distribution by the time $t+dt$, where dt is an elementary interval of time, with $0 < dt \ll 1,\ t+dt \le T_e$. It follows that a stochastic differential of a random process $X_C(t)$ can be introduced, namely the quantity $dX_C dt \overset{def}{\equiv} X_C(t+dt) - X_C(t)$, which designates a random distribution of the current stock of divisible production during the elementary interval of time dt.

Thus, taking into account the above specification of functions $S(t, x(t))$ and $C(t, x(t))$ the differential the equation (2) takes on form

$$dx(t) = S_{reg.}(t, x(t))dt + k_0 \cdot x(t) \cdot \delta(x(t), x_0(t))dt + dX_S - \\ -C_{reg.}(t)dt - k_1 \cdot x(t) \cdot \frac{1-\delta(x(t), x_1(t))}{x(t)+k_2}dt - \\ - \max_{t\in[T_s,T_e]} x_2(t) \cdot x(t) \cdot \frac{1-\delta(x(t), x_2(t))}{1+\dfrac{x_3(t)-x_2(t)}{x(t)-x_2(t)}}dt - dX_C. \qquad (4)$$

The following initial condition (5) must be added to (4):

$$x(t)\big|_{t=T_s} = x_s. \qquad (5)$$

The obtained equation (4) is the stochastic differential equation with respect to unknown random volume $x(t)$ of the current stock of divisible production; and this equation together with the initial condition (5) constitutes the Cauchy problem for determine required volume $x(t)$ of the current stock of divisible production.

It is significant that the summands dX_S and dX_C in the right-hand side of the equation (4) are not differentials in the usual sense; these summands must be understood in the sense of the Ito stochastic differential (Kuznetsov 2007). Besides, the indicator functions $\delta(x(t), x_i(t))$ $(i=\overline{0,2})$ in the right-hand side of the equation (4), derived according to formula (3), are not differentiated functions, which is caused by non-differentiability of the functions $S(t, x(t))$ and $C(t, x(t))$. Consequently, the requirement $\exists x'(t)\ \forall t \in [T_s, T_e]$, which was identified in the beginning of this section as a necessary condition for mathematical correctness of the model, will not be met. That is why in order to render a mathematical sense to the stochastic differential equation (4), it is necessary to introduce into is a corresponding amendment-condition. An easily realizable amendment might be substitution of the scalar functions $\delta(x(t), x_i(t))$ $(i=\overline{0,2})$ by the corresponding quadratic functions (which are smooth functions) on the sections $[0, x_i(t)]$ $(i=\overline{0,2})$, respectively. Such substitution is easily performed on the ground of natural and apparent requirements

$$\hat{\delta}(x(t),x_i(t))\Big|_{x(t)=0} = 1 \quad (i=\overline{0,2});$$

$$\hat{\delta}(x(t),x_i(t))\Big|_{x(t)=x_i(t)} = 0 \quad (i=\overline{0,2});$$

$$\int_0^{x_i(t)} \hat{\delta}(x(t),x_i(t))\,dx(t) = x_i(t) \quad (i=\overline{0,2});$$

and in the result the following differential functions are obtained:

$$\hat{\delta}(x(t),x_i(t)) = -\frac{3}{x_i^2(t)}\cdot x^2(t) + \frac{2}{x_i(t)}\cdot x(t) + 1 \quad \text{when}$$

$$x(t)\in[0,x_i(t)] \quad (\forall i=\overline{0,2}).$$

It is obvious that other substitutions-approximations are possible (for instance, by splines, etc.), which in comparison to the described above approach, i.e. approximation of scalar functions $\delta(x(t),x_i(t))$ $(i=\overline{0,2})$ by the corresponding smooth functions $\hat{\delta}(x(t),x_i(t))$ $(i=\overline{0,2})$ provide a higher level of precision. In this sense, there is certain ambiguity in determining the functions $\hat{\delta}(x(t),x_i(t))$ $(i=\overline{0,2})$, and hence ambiguity of the right-hand side of the equation (4).

Thus, instead of the differential equation (4) having no mathematical sense a mathematically correctly formulated differential equation can be written down:

$$dx(t) = S_{reg.}(t,x(t))dt + k_0\cdot x(t)\cdot\hat{\delta}(x(t),x_0(t))dt + dX_S -$$

$$-C_{reg.}(t)dt - k_1\cdot x(t)\cdot\frac{1-\hat{\delta}(x(t),x_1(t))}{x(t)+k_2}dt -$$

$$-\max_{t\in[T_s,T_e]} x_2(t)\cdot x(t)\cdot\frac{1-\hat{\delta}(x(t),x_2(t))}{1+\dfrac{x_3(t)-x_2(t)}{x(t)-x_2(t)}}dt - dX_C.$$

It is important to note the following with regard to the obtained stochastic differential equation. It is obvious that stochastic differentials of the random processes $X_S(t)$ and $X_C(t)$ can be conjoined if a random quantity $X(t)$ designating the total volume of productions that were delivered to and distributed from, the warehouse by the time t due to random circumstances. Then we can indeed determine a stochastic differential of the random process $X(t)$ as

$$dXdt \overset{def}{\equiv} X(t+dt) - X(t),$$

and this quantity will determine the change dynamics of the random volume of the divisible productions' stock during the elementary interval of time dt, namely $dXdt>0$ designates a random replenishment of stock during the elementary interval of time dt, and $dXdt<0$ designates a random distribution of stock during the elementary interval of time dt. With this specification in taken into account, the last differential equation takes the following final form:

$$dx(t) = S_{reg.}(t,x(t))dt + k_0\cdot x(t)\cdot\hat{\delta}(x(t),x_0(t))dt -$$

$$-C_{reg.}(t)dt - k_1\cdot x(t)\cdot\frac{1-\hat{\delta}(x(t),x_1(t))}{x(t)+k_2}dt -$$

$$-\max_{t\in[T_s,T_e]} x_2(t)\cdot x(t)\cdot\frac{1-\hat{\delta}(x(t),x_2(t))}{1+\dfrac{x_3(t)-x_2(t)}{x(t)-x_2(t)}}dt + dX. \qquad (6)$$

where $t\in[T_s,T_e]$; functions $S_{reg.}(t,x(t))$, $C_{reg.}(t)$ and $x_i(t)$ $(i=\overline{0,3})$, as well as numerical parameters k_i $(i=\overline{0,2})$ have the described above values and are viewed as the given initial data of the problem under consideration; the functions $\hat{\delta}(x(t),x_i(t))$ $(i=\overline{0,2})$ are determined by the following formulas:

$$\hat{\delta}(x(t),x_0(t)) = \begin{cases} 0, & if \quad x(t) > x_0(t), \\ -\dfrac{3}{x_0^2(t)}\cdot x^2(t) + \\ +\dfrac{2}{x_0(t)}\cdot x(t) + 1, & if \quad x(t)\le x_0(t); \end{cases} \qquad (7)$$

$$\hat{\delta}(x(t),x_1(t)) = \begin{cases} 0, & if \quad x(t) > x_1(t), \\ -\dfrac{3}{x_1^2(t)}\cdot x^2(t) + \\ +\dfrac{2}{x_1(t)}\cdot x(t) + 1, & if \quad x(t)\le x_1(t); \end{cases} \qquad (8)$$

$$\hat{\delta}(x(t),x_2(t)) = \begin{cases} 0, & if \quad x(t) > x_2(t), \\ -\dfrac{3}{x_2^2(t)}\cdot x^2(t) + \\ +\dfrac{2}{x_2(t)}\cdot x(t) + 1, & if \quad x(t)\le x_2(t). \end{cases} \qquad (9)$$

The stochastic differential equation (6) together with the initial condition (5), the initial given data $S_{reg.}(t,x(t))$, $C_{reg.}(t)$, $x_i(t)$ $(i=\overline{0,3})$ and k_i $(i=\overline{0,2})$, as well as approximating smooth indicator functions (7)-(9) is the Cauchy stochastic problem. It is a stochastic mathematical model for determining the current stock volume of divisible homogeneous production. Unfortunately, the given paper did not investigate the issue of finding an analytical solution of the constructed model (5)-(9). Nevertheless, as the following section will demonstrate, if we additionally require that the random process $X(t)$ will be the Markov random process, then the constructed continuous model (5)-(9) can be easily realized numerically (Ito 1987).

Remark 1. *Stochastic equation (6) shows that irrespective of the sign of the quantity* $x_s = x(t)\big|_{t=T_s}$ *(i.e. irrespective of the initial condition (5)), unknown* *function $x(t)$ can assume a negative value, which, at first sight, does not make any economic sense. But a possibility of such a case was purposefully taken into account prior to constructing mathematical model (5)-(9), and this case can be understood as a debt of the warehouse with regard to the current stock of divisible production. Besides, a closer look at the left-hand side of the equation (6) (as well as the equations (2) and (4)), it becomes obvious that there can be a case when $\frac{dx(t)}{dt}<0$, which means a negative rate if the quantity $\frac{dx(t)}{dt}$ is treated as the speed of a material point of the continuous medium in metric space, which has no physical sense. But if the quantity $\frac{dx(t)}{dt}$ in the considered problem designates the change rate of the volume $x(t)$ of the current stock at the time $t\in[T_s,T_e]$, then the case $\frac{dx(t)}{dt}<0$ corresponds to the situation whereby the volume $x(t)$ as a function of the time argument is a decreasing function, i.e. the accumulated stock of divisible productions in the warehouse is decreasing.*

3. CONSTRUCTION OF FINITE-DIFFERENCED MODEL FOR DETERMINATION OF RANDOM VOLUME OF DIVISIBLE HOMOGENEOUS PRODUCTION

In this section we offer a finite-differenced approximation of the mathematical model (5)-(9) for determination of current stock volume of divisible homogeneous production, which was constructed in the previous section. Besides, given some assumptions, we put forward a recurrent implicit differenced scheme for numeric determination of the random volume of divisible homogeneous production at given discrete moments of time.

Let us introduce the function

$$
f(t,x(t)) \overset{def}{\equiv} \begin{cases} S_{reg.}(t,x(t))+k_0\cdot x(t), & if \quad x(t)>x_0(t), \\ S_{reg.}(t,x(t))+\dfrac{3\cdot k_0}{x_0^2(t)}\cdot x^3(t)- \\ -\dfrac{2\cdot k_0}{x_0(t)}\cdot x^2(t), & if \quad x(t)\le x_0(t) \end{cases} -
$$

$$
-\begin{cases} \dfrac{1}{2}\cdot C_{reg.}(t,x(t))+k_1\cdot\dfrac{x(t)}{x(t)+k_2}, & if \quad x(t)>x_1(t), \\ \dfrac{1}{2}\cdot C_{reg.}(t,x(t))+\dfrac{3\cdot k_1}{x_1^2(t)}\cdot\dfrac{x^3(t)}{x(t)+k_2}- \\ -\dfrac{2\cdot k_1}{x_0(t)}\cdot\dfrac{x^2(t)}{x(t)+k_2}, & if \quad x(t)\le x_1(t) \end{cases} -
$$

$$
-\begin{cases} \max\limits_{t\in[T_s,T_e]} x_2(t)\cdot\dfrac{x(t)}{1+\dfrac{x_3(t)-x_2(t)}{x(t)-x_2(t)}}, \\ +\dfrac{1}{2}\cdot C_{reg.}(t,x(t)), & if \quad x(t)>x_2(t), \\ \dfrac{1}{2}\cdot C_{reg.}(t,x(t))+\dfrac{3\cdot\max\limits_{t\in[T_s,T_e]} x_2(t)}{x_2^2(t)}\cdot\dfrac{x^3(t)}{1+\dfrac{x_3(t)-x_2(t)}{x(t)-x_2(t)}}- \\ -\dfrac{2\cdot\max\limits_{t\in[T_s,T_e]} x_2(t)}{x_2(t)}\cdot\dfrac{x^2(t)}{1+\dfrac{x_3(t)-x_2(t)}{x(t)-x_2(t)}}, & if \quad x(t)\le x_2(t). \end{cases} \tag{10}
$$

Apparently, the function $f(t,x(t))$ is a random one since in its expression (10) the function of possible losses has been accounted

$$
C_{loss}(t,x(t),x_2(t),x_3(t))= \\ =\max_{t\in[T_s,T_e]} x_2(t)\cdot x(t)\cdot\frac{1-\delta(x(t),x_2(t))}{1+\dfrac{x_3(t)-x_2(t)}{x(t)-x_2(t)}},
$$

which were entered and determined at the previous stage. Without account of these losses the function $f(t,x(t))$ is an undetermined (i.e. non-random) function (Kopytov, Guseynov, Puzinkevich and Greenglaz 2010).

After introduction the function $f(t,x(t))$ on the formula (10) the stochastic equation (6) can be rewritten in a more compact way:

$$
dx(t)=f(t,x(t))dt+dX(t), \tag{11}
$$

and this equation is a particular instantiation (namely, $f_1(t,x(t))\equiv f(t,x(t))$; $f_2(t,x(t))\equiv 1$) of a more general stochastic differential equation in the Ito form

$$
dx(t)=f_1(t,x(t))dt+f_2(t,x(t))dX(t), \tag{12}
$$

where the functions $f_i(t,x(t))$ $(i=1,2)$ are supposed to be non-random functions, the random process $X(t)$ the Markov random process $X(t)$, and the quantity $dX(t)$ is understood in the sense of a stochastic differential Markov random process $X(t)$.

Under the mentioned assumptions, the Ito stochastic differential equation (12) allows for the following interpretation: for the stochastic differential $dX(t)$, which is contained in the right-hand side of the equation (12), the quantity $X(t)$ can be understood as a realized random quantity which assumes the given value $\tilde{x}=X(\tilde{t})$ at the moment $\tilde{t}\in[T_s,T_e]$. Moreover, due to the assumption that $X(t)$ is the Markov process the random quantity $X(\tilde{t}+dt)=\tilde{\tilde{x}}$, where $0<dt\ll 1$, $\tilde{t}+dt\le T_e$, has a density of probability $\rho(\tilde{\tilde{x}})=\rho(\tilde{t},\tilde{x};\tilde{t}+dt,\tilde{\tilde{x}})$. Then, if randomness of $\tilde{t}$ is

taken into account, the above speculation holds for $\forall t \in [T_s, T_e]$, i.e. for random $t \in [T_s, T_e]$, the random quantity $X(t+dt) = \tilde{\tilde{x}}$ where $0 < dt \ll 1,\ t+dt \le T_e$, is determined by the density of probabilities $\rho(\tilde{\tilde{x}}) = \rho(t, \tilde{x}; t+dt, \tilde{\tilde{x}})$ only if the random quantity $X(t)$ assumed the concrete value $\tilde{x}$ at the moment $t \in [T_s, T_e]$, i.e. if $X(t) = \tilde{x}$. This interpretation of the Ito stochastic differential equation (12) allows for rewriting the equation (12) in the finite-difference approximation, namely

$$x(t+\Delta t) - x(t) = f_1(t, x(t)) \cdot \Delta t + $$
$$+ f_2(t, x(t)) \cdot (X(t+\Delta t) - X(t)) =$$
$$= f_1(t, x(t)) \cdot \Delta t + f_2(t, x(t)) \cdot (X(t+\Delta t) - \tilde{x}).$$

If we accept

$$x(t) = f_2(t, x(t)) \cdot X(t) = f_2(t, x(t)) \cdot \tilde{x},$$

then we obtain a recurrent correlation

$$x(t+\Delta t) = f_1(t, x(t)) \cdot \Delta t + f_2(t, x(t)) \cdot X(t+\Delta t),$$

which can be used for a discrete definition of the value of unknown function $x(t)$. Indeed, if we break down the time segment $[T_s, T_e]$ into N elementary time spaces of the length Δt_i $(i = \overline{0, N-1})$ we will obtain the discrete mesh

$$\hat{T} \stackrel{def}{\equiv} \left\{ t_i: \ t_{i+1} = t_i + \Delta t_i \ \left(i = \overline{0, N-1}\right),\ t_0 = T_s,\ t_N = T_e \right\},$$

and after designating

$$x_i \stackrel{def}{\equiv} x(t_i),\ \tilde{x}_i \stackrel{def}{\equiv} X(t_i) = \frac{x(t_i)}{f_2(t_i, x(t_i))},$$

it is possible to write down the following recurrent implicit differenced scheme for determining the quantity $x(t)$ numerically:

$$x_{i+1} = f_1(t_i, x_i) \cdot \Delta t_i + f_2(t_i, x_i) \cdot \tilde{x}_{i+1}, \qquad (13)$$

where random quantities $\tilde{x}_{i+1}$ are determined by the density of probabilities

$$\rho\left(t_i, \frac{x_i}{f_2(t_i, x_i)}; t_{i+1}, \tilde{\tilde{x}}\right).$$

Remark 2. *Mathematical model (5)-(9) constructed in the Section 2 can be solved analytically with the help of integrals of Stratonovich and Ito (see (Kuznetsov 2007)) assuming a Markov nature of the random process $X(t)$. If this assumption is not made (or can not be made due to specificity of the particular task of inventory control), the question of how to analytically integrate the stochastic differential equation (9) remains, unfortunately, still open, and as mentioned before research into this issue was not undertaken in the present paper. As shown in (Diend 1990), though, with certain additional conditions but without assuming the Markov nature of the random process $X(t)$ an effective approximation of a stochastic differential equation such as (12), particularly the equation (11), which is the equation (6) in the mathematical model (5)-(9) constructed in the Section 2.*

Remark 3. *The constructed recurrent differenced scheme (13) together with the initial condition (5) is a finite differenced mathematical model for defining one of possible trajectories of the random quantity $x(t)$, i.e. the constructed finite differenced model (13); (5) allows for defining approximate values of the quantity $x(t)$ at the moments of time t_i: $t_{i+1} = t_i + \Delta t_i$ $\left(i = \overline{0, N-1}\right)$, $t_0 = T_s$, $t_N = T$.*

4. STOCHASTIC CONTINUOUS MODELS FOR SIMULTANEOUSLY DEFINING VOLUMES OF CURRENT STOCK OF DIVISIBLE PRODUCTION AT SEVERAL INTERCONNECTED WAREHOUSES

The present section suggests two stochastic continuous mathematical models for defining volumes of current stock of divisible homogeneous and heterogeneous productions at several interconnected warehouses simultaneously. For achieving this aim, similarly to the Section 2, apparatus of mathematical physics is used and principle of continuous medium, the language of the theory of partial differential equations is chosen as a modeling language. Because of paper's space limitations there is, unfortunately, no opportunity to present the entire chain of argumentation and all calculations related to constructing these models "from scratch"; they are only mathematically represented in what follows, with minimal explanation. Also, it is necessary to underline that in the given section some definitions introduced and employed in the three previous sections, the definitions $x_i(t)$ $(i = 1, 2)$, in particular, have different values which are being declared as soon as they are introduced.

So, $m \in \mathbb{N}$ warehouses are under consideration, and it is assumed that dynamics of the volume of divisible homogeneous production in all m warehouses is subject to the stochastic differential equation (6) which was obtained in the Section 2. For the stochastic differential $dX(t)$ that is contained in the right-hand side of the equation (6), the quantity $X(t)$ will be viewed as a realized random quantity which assumed the given value $\tilde{x} = X(t)$ at the time $t \in [T_s, T_e]$, i.e. the given warehouse has the volume of divisible homogeneous production $\tilde{x} = x(t)$ at the fixed time $t \in [T_s, T_e]$; as the course of constructing equation (6) shows, this volume can comprise both determined and random constituent volumes. Then the random quantity $X(t+dt) = \tilde{\tilde{x}}$, where $0 < dt \ll 1,\ t+dt \le T_e$, designates a random volume of homogeneous production in a particular warehouse at the moment $t+dt$ under the condition that the volume $\tilde{x}$ of homogeneous production was present in this very warehouse at the previous moment t. Consequently, it can be said that the random quantity $X(t+dt)$ has the density of probability $\rho(\tilde{\tilde{x}}) = \rho(t,\tilde{x};t+dt,\tilde{\tilde{x}})$.

Since the continuous mathematical model (5)-(9) constructed in the Section 2 assumed the existence of one warehouse where there was volume $x_s = x(t)|_{t=T_s}$, of divisible homogeneous foods at the initial moment of time $t = T_s$, for m interconnected warehouses there are obviously m initial conditions

$$x^{\{i\}}(t)\Big|_{t=T_s} = x_s^{\{i\}},\ \ i = \overline{1,m},$$

where $x^{\{i\}}(t)$ designates a random volume of the divisible homogeneous products in an i-warehouse at the time $t \in [T_s, T_e]$. That is why once these random initial volumes $x_s^{\{i\}},\ i = \overline{1,m}$ were distributed on the axis OX of the Cartesian rectangular system of coordinates, these irregularly distributed initial volumes can be mentally identified with the distribution of the warehouses on the axis OX. This identification allows for constructing the required mathematical model. It is worth mentioning here that topology of the imagined distribution of warehouses on the axis OX does not have to match the typology of distributing initial quantities-volumes $x_s^{\{i\}},\ i = \overline{1,m}$; this is natural and obvious.

After the above mentioned identification we have a certain set of interconnected warehouses (SIW), and we can construct a mathematical model for establishing the dynamics of random volumes of divisible homogeneous production in this SIW ignoring the dynamics of a random volume of divisible homogeneous production in any individual warehouse.

Let us consider a relatively short segment $[x(t), x(t)+\Delta x(t)]$ of the length $\Delta x(t)$ and introduce the functional $\Delta\Psi(t,x(t))$ of the function- volume $x(t)$ which describes the number of elements SIW that can be found in the segment $[x(t), x(t)+\Delta x(t)]$. In other words, $\Delta\Psi(t,x(t))$ is the number of warehouses distributed on a short segment $[x(t), x(t)+\Delta x(t)]$ of the length $\Delta x(t)$. Then $\dfrac{\Delta\Psi(t,x(t))}{\Delta x(t)}$ can be treated as probability of the warehouse with the volume $x(t)$ of production being on the segment $[x(t), x(t)+\Delta x(t)]$. Consequently, we can move over to the limit with $\Delta x(t) \to 0$ and define a new function

$$p(t,x(t)) \overset{def}{\equiv} \lim_{\Delta x(t)\to 0} \frac{\Delta\Psi(t,x(t))}{\Delta x(t)},$$

which is the density of distribution of warehouses according to random volumes $x(t)$ of divisible homogeneous production. Then the function

$$\Psi(t) \overset{def}{\equiv} \int_{x_1}^{x_2} p(t,x(t))\,dx(t)$$

designates the number of warehouses with random volumes $x(t) \in [x_1(t), x_2(t)]$ at the time moment $t \in [T_s, T_e]$.

It is easily seen that

$$\int_{T_s}^{T_e} \Psi(t)\,dt \equiv m;$$

$$\int_{-\infty}^{+\infty} p(t,x(t))\,dx(t) \equiv 1.$$

Now the density of distribution $p(t,x(t))$ of warehouses according to random volumes $x(t)$ of divisible homogeneous production is defined, and we can establish the law of distributing warehouses according to random volumes, i.e. to find out the rule that governs the change of the function $p(t,x(t))$. For this, the axis OX is divided into two parts, an arbitrary segment $[x_1(t), x_2(t)]$ and the view of this segment, i.e. the domain $(-\infty, x_1(t)) \cup (x_2(t), +\infty)$. As random volumes of productions in warehouses change with the course of time, it will mean in our case that warehouses will be moving along the axis OX in this course of time. This, in turn, means that during the segment of time $[t_1, t_2],\ \forall t_1, t_2 \in [T_s, T_e]$ a certain number of warehouses will have random volumes of divisible homogeneous production that are no bigger than $x_1(t)$ and no less than $x_2(t)$, i.e. some warehouses will be located in the segment $[x_1(t), x_2(t)]$ whereas their remaining number will be outside this segment, or in the domain $(-\infty, x_1(t)) \cup (x_2(t), +\infty)$. Thus it will be quite correct if the equation of balance of warehouses for the segment $[x_1(t), x_2(t)]$ in the segment of time $[t_1, t_2]$ is presented in the following way (on analogy with a widely known approach in mathematical physics whereby mathematical models are constructed for heat conductivity, waves, diffusion, radiation, and other physical processes):

$$\Delta\Psi(t_1,t_2) \overset{def}{\equiv} \Psi(t_2) - \Psi(t_1) = \Psi^{\{1\}} + \Psi^{\{2\}} + \Psi^{\{3\}},$$

where $\Psi^{\{1\}}$ is the number of warehouses located in the segment $[x_1(t), x_2(t)]$ in the segment of time $[t_1, t_2]$ due to non-random replenishments and distributions of divisible homogeneous production; $\Psi^{\{2\}}$ is the number of warehouses located in the segment $\left[x_1(t), x_2(t)\right]$ in the segment of time $[t_1, t_2]$ due to random replenishments and distributions of divisible homogeneous production; $\Psi^{\{3\}}$ is the number of warehouses which get in the period $\left[x_1(t), x_2(t)\right]$ or get out of it in the period of time $[t_1, t_2]$ for the account of random losses (both normal/predicted and abnormal/extraordinary which were discussed in section 2) current stocks of the divisible homogenous production. The function $\Delta\Psi(t_1, t_2)$ in the left-hand side of (13) is calculated according to the formula

$$\Delta\Psi(t_1,t_2) \overset{def}{\equiv} \Psi(t_2) - \Psi(t_1) =$$

$$= \int_{x_1}^{x_2} p(t_2, x)\,dx - \int_{x_1}^{x_2} p(t_1, x)\,dx =$$

$$= \int_{x_1}^{x_2} p(t, x(t))\Big|_{t=t_1}^{t=t_2} dx = \int_{x_1}^{x_2} dx \int_{t_1}^{t_2} \frac{\partial p(t, x(t))}{\partial t} dt. \qquad (14)$$

It is obvious that the quantities $\Psi^{\{i\}}$ $\left(i=\overline{1,3}\right)$ can be negative, and this is then treated as a removal of warehouses from the segment $\left[x_1(t), x_2(t)\right]$. The final formulas for the functions $\Psi^{\{i\}}$ $\left(i=\overline{1,3}\right)$ are given below without conclusion (there is an elegant conclusion which is not given here due to the space constraints):

$$\Psi^{\{1\}} = \int_{t_1}^{t_2} \left\{ p(t, x(t)) \cdot \vartheta(t, x(t)) \right\} \Big|_{x(t)=x_2(t)}^{x(t)=x_1(t)} dt =$$

$$= \int_{t_1}^{t_2} dt \int_{x_1}^{x_2} \frac{\partial}{\partial x(t)} \left(p(t, x(t)) \cdot \vartheta(t, x(t)) \right) dx(t), \qquad (15)$$

$$\Psi^{\{2\}} = \int_{t_1}^{t_2} dt \int_{x_1}^{x_2} \left\{ -\frac{\partial}{\partial x} \left(a(x(t), t) \cdot p(t, x(t)) \right) + \right.$$

$$\left. + \frac{1}{2} \cdot \frac{\partial^2}{\partial x^2} \left(b(x(t), t) \cdot p(t, x(t)) \right) \right\}, \qquad (16)$$

$$\Psi^{\{3\}} = \int_{t_1}^{t_2} dt \int_{x_1}^{x_2} L(t, x(t))\,dx(t), \qquad (17)$$

where function $L(t, x(t))$ is the number of warehouses which for a single period of time, caused only by the random losses of the current stocks of the divisible homogenous production, will be moved to the single segment of the abscises OX, at which there are distributed the values of random volumes of all m warehouses at the moment of time $t \in [T_s, T_e]$; the function $\rho(z, s; x, t)$ is a transitional function of the probability density of a diffusion stochastic process $X(t)$ (Samarsky and Mikhailov 2002; Gikhman and Skorokhod 1982); the function $\vartheta(t, x(t))$ designates the change rate of the random volume $x(t)$ of the current stock of divisible homogeneous production in the set of interconnected warehouses (SIW) at the time t, and is determined by the stochastic equation

$$\vartheta(t, x(t)) = \frac{dx(t)}{dt} = S(t, x(t)) - C(t, x(t)),$$

where the functions $S(t, x(t))$ and $C(t, x(t))$ have the same values as mentioned in the Section 2. The functions $a(x(t), t)$ and $b(x(t), t)$ are calculated by the formulas

for $\forall \varepsilon > 0,\ \forall z \in \mathbb{R}^1$ $\quad a(t, x(t)) =$

$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \cdot \int_{|x(t)-z(t)| \le \varepsilon} \left(x(t) - z(t) \right) \cdot \rho\left(x(t), t; z(t), t + \Delta t \right) dz(t),$$

for $\forall \varepsilon > 0,\ \forall z \in \mathbb{R}^1$ $\quad b(t, x(t)) =$

$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \cdot \int_{|x(t)-z(t)| \le \varepsilon} \left(x(t) - z(t) \right)^2 \cdot \rho\left(x(t), t; z(t), t + \Delta t \right) dz(t).$$

Taking into account expressions (15)-(17) in formula (14), the following equation is obtained:

$$\int_{x_1}^{x_2} dx \int_{t_1}^{t_2} \frac{\partial p(t, x(t))}{\partial t} dt =$$

$$= \int_{t_1}^{t_2} dt \int_{x_1(t)}^{x_2} \frac{\partial}{\partial x(t)} \left(p(t, x(t)) \cdot \vartheta(t, x(t)) \right) dx(t) +$$

$$+ \int_{t_1}^{t_2} dt \int_{x_1}^{x_2} \left\{ -\frac{\partial}{\partial x} \left(a(x(t), t) \cdot p(t, x(t)) \right) + \right.$$

$$\left. + \frac{1}{2} \cdot \frac{\partial^2}{\partial x^2} \left(b(x(t), t) \cdot p(t, x(t)) \right) \right\} + \int_{t_1}^{t_2} dt \int_{x_1}^{x_2} L(t, x(t))\,dx(t),$$

which due to arbitrariness of the selected volume segment $\left[x_1(t), x_2(t)\right]$, arbitrariness of the selected time segment $[t_1, t_2]$, and in accordance with the First Mean Value Theorem (use of this theorem here is quite rightful because all its requirements are met) can be written in the following way:

$$\frac{\partial p(t, x(t))}{\partial t} = -\frac{\partial}{\partial x} \left(\left[a(t, x(t)) + \vartheta(t, x(t)) \right] \cdot p(t, x(t)) \right) +$$

$$+ \frac{1}{2} \cdot \frac{\partial^2}{\partial x^2} \left(b(t, x(t)) \cdot p(t, x(t)) \right) + L(t, x(t)). \qquad (18)$$

The resulting stochastic equation (18) is the parabolic type inhomogeneous particular differential equation, and together with the above mentioned functions $a(t, x(t)) \not\equiv 0$, $b(t, x(t)) \not\equiv 0$ and $\vartheta(t, x(t))$, as well as corresponding initial and boundary conditions (for instance, the Newton type boundary conditions, or Neumann boundary conditions, or non-located boundary conditions) it makes the required mathematical model for determining unknown density of distribution $p(t, x(t))$ of exactly $m \in \mathbb{N}$ warehouses according to random volumes $x(t)$ of divisible homogeneous production. It is not difficult to see that the equation (18) is a particular case of the widely known equation of Kolmogorov for the Markov stochastic process $X(t)$ with a transition function of the density of probability $\rho(z,s;x,t)$.

The next stochastic continuous model (with the Dirichlet boundary conditions) is an informal generalization (the corresponding conclusion is rather complex and therefore not presented in the given article) of the above mentioned model: it describes the dynamics of unknown density of distribution $p(t,x_1(t),...,x_n(t))$ of exactly $m \in \mathbb{N}$ warehouses according to random volumes $x(t)=(x_1(t),...,x_n(t))$ of divisible $n \in \mathbb{N}$ heterogeneous productions

$$\frac{\partial p(t,x(t))}{\partial t}=\frac{1}{2}\cdot\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2}{\partial x_i \partial x_j}\left(b_{ij}(t,x(t))\cdot p(t,x(t))\right)-$$

$$-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left(\left[a_i(t,x(t))+\vartheta_i(t,x(t))\right]\cdot p(t,x(t))\right)+$$

$$+\sum_{i=1}^{n}L_i(t,x(t)),$$

$$p(t,x(t))\Big|_{t=T_s}=p_0(x_1,...,x_n),\ x_i\in\mathbb{R}^1\ \ \forall i=\overline{1,n};$$

$$p(t,x(t))\Big|_{x_i(t)=l_i^{\{1\}}-0}=p_i^{\{1\}}(t),\ l_i^{\{1\}}\in\mathbb{R}^1\ \ \left(i=\overline{1,n}\right);$$

$$p(t,x(t))\Big|_{x_i(t)=l_i^{\{2\}}-0}=p_i^{\{2\}}(t),\ l_i^{\{2\}}\in\mathbb{R}^1\ \ \left(i=\overline{1,n}\right),$$

where

$$x(t)=(x_1(t),...,x_n(t))\in\bigcup_{i=1}^{n}\left[l_i^{\{1\}},l_i^{\{2\}}\right];$$

the function $x_i(t)$ $\left(i=\overline{1,n}\right)$ describes the random volume of i-th divisible production at the time moment $t\in[T_s,T_e]$; the function $\vartheta_i(t,x(t))$ describes the change rate of the random volume $x_i(t)$ of the current stock of i-th divisible production in the set $m\in\mathbb{N}$ of interconnected warehouses at the time moment t; the functions $a_i(x(t),t)$ $\left(i=\overline{1,n}\right)$ and $b_{ij}(t,x(t))$, $\left(i=\overline{1,n};\ j=\overline{1,n}\right)$ are calculated according to the formulas

$$a_i(t,x(t))=$$

$$=\lim_{\Delta t\to 0}\frac{1}{\Delta t}\cdot\int_{B_\varepsilon(z(t))}\left(x_i(t)-z_i(t)\right)\cdot\rho(x,t;z,t+\Delta t)\,dz(t),$$

$$b_{ij}(t,x(t))=$$

$$\lim_{\Delta t\to 0}\frac{1}{\Delta t}\cdot\int_{B_\varepsilon(z(t))}\left(x_i(t)-z_i(t)\right)\cdot\left(x_j(t)-z_j(t)\right)\cdot dz(t),$$

where

$$z(t)\overset{def}{\equiv}\{z_1(t)...z_n(t)\};\ \ x(t)\overset{def}{\equiv}\{x_1(t),...,x_n(t)\};$$

the function $\rho(x,t;z,t+\Delta t)$ is a transition function of the density of probabilities of the diffusion stochastic process $X(t)\overset{def}{\equiv}(X_1(t),...,X_n(t))$ (Samarsky and Mikhailov 2002; Gikhman and Skorokhod 1982), and $B_\varepsilon(z(t))\overset{def}{\equiv}\left\{x(t):\ \|x(t)-z(t)\|_{\mathbb{R}^n}\le\varepsilon\right\}$ is the closed ε-neighborhood of the point $z(t)$.

CONCLUSIONS

The present paper studies construction of unsteady stochastic ODE and PDE models for calculating the volume of current stock of divisible productions "from scratch" using apparatus of mathematical physics. The constructed models are new ones and can be used for on-line monitoring of the dynamics of the divisible productions random volumes. Further guidelines of the current research are the development and investigation of optimization inventory control tasks using cost criteria on the base of on-line monitoring of multiproduct stock of divisible productions in one or in several interrelated warehouses.

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