A 256 VARIABLE NONLINEAR TRANSPORTATION PROBLEM

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ABSTRACT

The classic mathematical transportation problem which is presented in most operations research or industrial engineering courses involves shipping from a number of warehouses to a number of customers, where the shipping costs are linear. The idea being if it costs one euro dollar to ship one unit, from location A to location B, it will cost 100 euro dollars to ship 100 units from warehouse A to customer B. These linear assumptions (for all the shipping costs) are usually made so that various linear programming techniques (simplex and other linear approaches) can be used to efficiently solve the overall optimization problem. However, in the practical business world of large scale shipping there are usually considerable returns to scale (nonlinear costs) associated with the deliveries to customers. Therefore, an example of this kind is presented here and solved using simulation techniques.

Keywords; nonlinear transportation problem, Monte Carlo

1. INTRODUCTION

Specifically the example presented involves shipping a product from 16 warehouses to 16 preferred customers where each of the 16X16=256 shipping routes has a different nonlinear cost structure, which reduces the unit shipping costs from warehouse I to customer J as the number of units shipped increases.

The example here has between 525,000 and 600,000 units (in various amounts) of the product stored in the 16 locations and the customer demands are for between 525,000 and 600,000 units (in different amounts) at each destination. Also, the total supply equals the total demand in this case (which is usually true in the long run). However, the supply rarely equals short term demand in the real world and the MSMCO simulation approach can be modified to deal with some less than or greater than constraints rather than equations, if supplies and demands are not balanced.

2. THE SAMPLE CASE STUDY

We define 256XIJ variables, where XIJ is the amount of product leaving a warehouse I and heading to customer J. We also have 16 equations for the XIJ values leaving each warehouse, plus 16 equations for the XIJ values arriving at the 16 preferred customers in the correct amounts they ordered. Additionally, we have a nonlinear cost equation. Although it may cost one euro dollar to ship one unit of product, it may cost a lot less than 100 euro dollars to ship 100 units. Therefore, the 256 variable nonlinear transportation problem (with discounts for bulk shipping) is solved with the multi stage Monte Carlo optimization (Conley, 2003) simulation technique. The shipping company's goal is to reduce the overall shipping costs that were running in the 1.5 to 2 million euro dollars range down to about one million euro dollars. Therefore, the right hand side of the cost equation is set at one million euro dollars. The other 32 equations are in units of product shipped. We use multi stage Monte Carlo optimization (MSMCO) to try to minimize the sum of the absolute values of the differences between the left and right hand side of the 33 equations. This will approximately solve the system. The multi stage Monte Carlo system makes repeated solution attempts in an ever moving and decreasing in size feasible solution region following a trail of better and better answers (lower total error) though 257 dimensional spaces. The cost equation is C=ΣΣ7.+.05*(i+j)*(x(i+j))**(.6+.01*(i+j)) = 1000000 where i=1,2, …16 and * is multiply and ** is raise to a power.

Specifically, for our 33 equation 256 variable system (transformed into a 257 dimensional optimization problem) 40,000 random sample answers are looked at in stage one and the best one (stored and printed) had a total error in all 33 equations of 119,996,528. Then stage two looked at another 40,000 sample answers in a reduced region initially centered at the best answer from stage one. This produced a new best answer of 114,673,528. Then stage three similarly does another 40,000 sample solutions reducing the error further.

Even though these initial "discovery" stages have very high total errors, by stage 50 the total error is down to 34. The 256XIJ values produced in stage 50 solved all 32 of the warehouse and customer equations and had a 34 euro dollar error in the cost control equation where the goal was a cost of one million euros.

This simulation took about 3 minutes to run the 50 x 40,000 = 2,000,000 function evaluations on an inexpensive desk top PC. The complete printout of the total errors of the 50 stage MSMCO (multi stage) simulation is in Table 1. Tables 2 and 3 present the other relevant parts of the answer.

3. THE ANSWER PRINTOUTS

Table 1 (as mentioned) presents the 50 stage printout of the total errors. They are very high in the early discovery stages of the MSMCO simulation run. However, once they start to track a good answer the error terms start dropping dramatically to a 33+ Euro dollar cost error and a virtual zero, error (.5 unit total) on all of the 32 leaving and arriving equations. The tables follow here.

Table 1: The Stage Errors

Stage Number	Total Errors
1	119996528.0000
2	114673528.0000
3	101945352.0000
4	93896400.0000
5	84041472.0000
6	67086016.0000
7	51818592.0000
8	30441744.0000
9	18462332.0000
10	10419225.0000
11	4611007.0000
12	1350197.2500
13	829315.1875
14	470044.2813
15	235782.6875
16	158225.0000
17	85789.7500
18	69592.3125
19	44407.7500
20	36262.8125
21	20031.7500
22	14905.0625
23	10706.0625
24	7548.3125
25	4448.8750
26	3890.5625
27	2486.1250
28	1994.3125
29	1264.9375
30	909.5000
31	714.6875
32	516.1250
33	409.8125
34	300.6250
35	229.2500
36	150.5000
37	120.0000
38	99.5000
39	79.3125
40	64.7500
41	58.4375
42	47.9375
43	44.8125
44	40.5000
45	39.1250
46	37.8125
47	36.6250
48	36.2500
49	35.1250
50	33.0625

Table 2 presents the number of units shipped from each warehouse to each customer. The 16 numbers in the 1 grouping represent the unit shipping amounts leaving warehouse 1 and bound for customers 1 through 16 (reading left to right top to bottom in the grouping labeled one). They add up to 600,000 units. The 16 numbers in the 2 groupings represent the unit shipping amounts leaving warehouse 2 and bound for customer 1 through 16 (reading left to right top to bottom in the grouping labeled two). They add up to 595,000 units (and so on for the other 14 warehouses).

Taking the upper left hand entry in each of the 16 groupings and adding them up gives you the 525,000 total units bound for customer 1 that come from warehouses 1 through 16 (and so on).

Table 2: The Shipping Amounts

	Column 1	Column 2	Column 3	Column 4
1	44592.266	28204.779	24779.855	38818.793
1	44660.852	42474.914	36898.750	8789.456
1	24376.973	32607.883	44095.602	35170.844
1	47782.434	107502.773	32258.361	6985.296
2	860.316	4483.140	52095.824	4920.257
2	55721.340	41307.066	19379.221	13734.656
2	15755.903	10778.478	37142.035	84397.047
2	134475.094	33806.871	67846.141	18296.627
3	14016.597	82811.031	33526.914	3709.981
3	2007.490	14209.472	475.354	57077.633
3	60822.930	65550.570	14076.178	48264.734
3	49168.039	10770.127	104711.500	28801.520
4	55947.031	50030.691	134804.719	45923.613
4	12304.259	16243.791	28020.822	2435.157
4	10291.113	78165.891	36343.551	1473.928
4	28673.438	3491.757	38109.355	42740.895
5	2080.985	12592.176	105.309	166457.266
5	17227.412	40241.980	29042.160	14684.987
5	25134.654	10384.798	57884.641	64610.441
5	31199.455	32094.121	23535.330	52724.262
6	1486.628	27140.459	36218.969	15398.490
6	2398.268	38748.012	45646.762	17540.447
6	14546.153	52363.535	38594.148	28070.598
6	3121.571	86585.625	33644.195	133496.156
7	266746.000	17605.098	9369.590	15545.590
7	11775.452	47188.070	30555.365	18927.512
7	8619.394	15634.977	921.640	55111.660
7	888.766	4657.088	61295.070	5158.753
8	16740.145	4613.849	15247.325	986.281
8	30984.273	22648.006	105562.391	5193.723
8	11082.579	112332.195	72157.844	72018.586
8	5704.437	28591.939	30635.119	30501.340
9	23591.436	2920.417	83545.125	23442.521
9	71350.711	7149.650	6071.913	28681.975
9	105054.766	29472.561	1399.122	25688.439
9	98983.820	1544.764	22257.094	28845.840
10	2816.591	9420.608	17896.875	38675.965
10	80396.500	30036.746	88567.492	3042.466
10	31810.510	4844.226	153020.750	13448.157

10	5353.755	70741.266	94.583	4833.566
11	17829.125	81727.445	16869.154	6623.472
11	13223.104	13930.215	13169.904	8801.730
11	117752.563	34185.121	33070.383	12759.497
11	22923.820	58233.043	32604.734	66296.680
12	4566.582	19541.547	2702.872	7670.750
12	39572.512	48614.906	13704.979	162585.531
12	54835.453	4228.451	10454.256	3485.226
12	89607.859	46765.195	14803.914	21860.117
13	7547.889	15834.984	4474.897	14508.642
13	64135.563	151853.094	12974.294	74015.195
13	16960.918	307.568	23610.848	18164.500
13	18629.258	8608.676	45110.926	63262.758
14	10455.435	103111.766	1596.627	113677.133
14	27811.521	6396.921	6037.331	26470.602
14	5542.942	57488.332	30804.586	65985.578
14	1965.880	18959.227	24096.121	34599.902
15	29200.139	57121.152	59628.168	27809.139
15	10280.215	28387.352	96994.922	34976.105
15	47973.891	420.451	12072.192	37972.953
15	11927.979	30296.170	19933.180	25006.080
16	26522.867	12840.830	42137.781	15832.064
16	61150.555	569.841	21898.381	83042.867
16	14439.232	61235.004	9352.340	13377.782
16	34594.410	47351.375	44064.352	36590.328

Table 3 gives the individual equation errors for the 32 leaving and arriving units followed by the cost equation error 33+ Euros in the left column. The right hand column gives the right hand side constants on all 33 equations.

Table 3: Error on Left Constants on Right

Equation Errors	Right Hand Side
0.00000	525000.00000
0.00000	530000.00000
0.00000	535000.00000
0.00000	540000.00000
0.00000	545000.00000
0.00000	550000.00000
0.00000	555000.00000
0.00000	560000.00000
0.00000	565000.00000
0.00000	570000.00000
0.06250	575000.00000
0.00000	580000.00000
0.00000	585000.00000
0.00000	590000.00000
0.00000	595000.00000
0.06250	600000.00000
0.12500	600000.00000
0.00000	595000.00000
0.00000	590000.00000
0.00000	585000.00000
0.00000	580000.00000
0.00000	575000.00000
0.00000	570000.00000
0.00000	650000.00000
0.06250	560000.00000
0.00000	555000.00000
0.00000	550000.00000
0.06250	545000.00000
0.00000	540000.00000
0.06250	535000.00000
0.00000	530000.00000
0.00000	525000.00000
33.06250	1000000.00000

4. LINEAR VERSUS NONLINEAR SHIPPING COSTS

Linear programming was developed in the early to mid 20th century as a result of the well-developed theory of linear algebra for solving systems of equations. It involves solving linear equations and/or inequalities while heading to the goal of optimizing an objective function subject to linear constraints (equations and inequalities). The key result that made this type of multivariate linear optimization possible is the fundamental theorem of linear programming which states that the optimal solution is at a "corner point" in the feasible solution space. This solution technique is great for small or large scale (number of variables) shipping problems that are truly linear in nature or can at least be reasonably approximated with a linear system.

The difficulty of course is that many shipping problems are multivariate and nonlinear with constraints and these are more difficult to solve. A fundamental theorem of nonlinear programming (if it existed) would say that the optimal solution could be anywhere in the feasible solution space (at a corner point or in the interior of the feasible solution spaces).

That is why simulation based multivariate nonlinear optimization techniques such as multi stage Monte Carlo optimization (MSMCO) can be useful when other techniques do not work or are not available.

5. MULTI STAGE MONTE CARLO OPTIMIZATION

The multi stage (MSMCO) simulation technique randomly looks around the entire feasible solution space in stage one and samples several thousand feasible solutions and stores and prints the best answer so far. That is the traditional Monte Carlo (or random) optimization technique. However, with MSMCO that is just stage one. Then centered about this best answer so far, stage two looks at thousands more feasible solutions in a slightly reduced search region and stores and prints its "best answer" so far. Then stage three in a slightly more reduced region centered about the stage two best answers repeats this process. Our particular example here did a 50 stage MSMCO simulation. It started with huge error terms in the early stages. However, by stage 50 it had produced a useful answer to a 33 by 256 nonlinear system.

This type of program can be run quickly on the modern inexpensive desk top computer available in our 21st century. Some additional applications of multi stage Monte Carlo optimization (MSMCO) to various shipping problems are in (Conley 2003). More general applications of the MSMCO technique are in (Wong 1996) and (Conley 2008). It is a fairly versatile approach to general nonlinear optimization problems. The problems are more difficult to solve as the number of variables increases. However, computer speeds help with that difficulty.

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William Conley received a B.A. in mathematics (with honors) from Albion College in 1970, an M.A. in mathematics from Western Michigan University in 1971, a M.Sc. in statistics in 1973 and a Ph.D. in mathematics-computer statistics from the University of Windsor in 1976. He has taught mathematics, statistics, and computer programming in universities for over 30 years. He is currently a professor emeritus in Business Administration and Statistics at the University of Wisconsin at Green bay. The developer of multi stage Monte Carlo optimization and the CTSP multivariate correlation statistics, he is the author of five books and 203 publications world wide. He is a member of the American Chemical Society, a fellow in the Institution of Electronic and Telecommunication Engineers and a senior member of the Society for Computer Simulation.