DYNAMIC MODELING OF A PARALLEL ROBOT WITH SIX DEGREES OF FREEDOM

A. Chibani[a] ; **C. Mahfaudi**[b] ; **A. Zaatri**[c] ; **A. Amouri**[d]

[a] , [b] , [d] University Larbi Ben M'hidi Oum El Bouaghi, Faculty of Science and Technology.

[c] University Mantouri of Constantine.

[a]: chibaniabdelhakim@yahoo.fr
[b]: c_mahfoudi_dz@yahoo.fr
[c]: azaatri@yahoo.com

ABSTRACT

Parallel robots seem to be the most suitable spots requiring high performance such as speed and accuracy. Such performances sought now are that the dynamics of parallel structures is no longer negligible.
This work represents a contribution in this latter context; it deals in the whole dynamic study of a parallel robot with six degrees of freedom constituting the so-called Gough Stewart platform. In determining the direct and inverse geometric model, we use a setup based on Khalil and Kleinfinger ratings [7] for structures with closed loops. The kinematical modeling, using the calculation of the Jacobian matrix and its inverse, were deduced from the joint velocities of the six cylinders in order to follow a desired trajectory for the platform.
The Newton Euler formalism is used to model the dynamics of the robot and the first to consider each kinematics chain (legs) as a serial structure, and then by considerations of balance and closed chain, we determine the dynamics of the platform.

Keywords: Gough-Stewart platform, parallel robot, dynamic modeling, Newton Euler formalism.

1. INTRODUCTION

The complex architecture of parallel robots makes it increasingly necessary to improve their dynamic performance, this is especially motivated by the proven qualities recognized by the research community [2] and industrial robotics, addressing the very high speed, driving simulators, machine - tools, medical applications, etc.. Parallel robots in comparison with serial robots, have special characteristics, stiffness and dynamic load capacity higher still, actuators, high precision guidance and stable operation.
To obtain the dynamics of parallel robots, many methods have used the classical procedure of computing dynamic model of an equivalent tree structure; Principle of virtual works has been used in [4]; work [3] and [5] have used the Euler-Lagrange formalism. On the other hand, the equations of Newton - Euler have been used by [10], [13] and [14]. Through the establishment of iterative matrix relations for kinematics and dynamic analysis of parallel robot Gough - Stewart [17] used the principle of virtual work to derive the fundamental equations of dynamics.
This paper presents a method for obtaining the inverse dynamic model of parallel robot with six degrees of freedom consisting of a mobile platform attached to a fixed base by six identical kinematics chains using an universal joint (on base), a spherical joint (on the mobile platform) and an active prismatic joint. The model based on the formalism of Newton – Euler is obtained in terms of the dynamic models of the legs. It concludes with a simulation of kinematics and dynamics of the robot.

2. DESCRIPTION OF PARALLEL ROBOT.

The proposed parallel robot to look like a platform composed of six identical kinematics chains linked to a mobile platform through ball and connected with a fixed base by universal joints. A prismatic actuator used to vary the length of the kinematics chains (Fig. 1), the platform provides six degrees of freedom. Geometric modeling, kinematics and dynamics of parallel robots require the knowledge descriptions robots closed structure, description of tree structure robots as well as open-chain robots. It defines two headers one R_0 attached to base its origin is the point O, the other set at R_p mobile platform with O_2 as the origin.

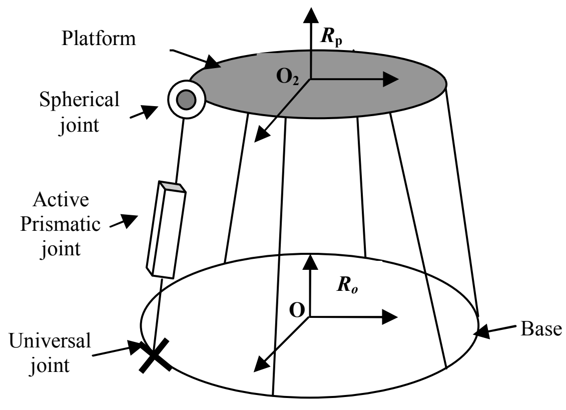

Fig. 1: Description of parallel robot with six degrees of freedom

The center of each drive shaft and the center of each ball are denoted by Bi and Pi (i = 1 to 6).

The robot consists of 5 loops space motorized prismatic joints 6 and 30 passive revolute joints. The tree structure equivalent minimum [3] is obtained by isolating the platform. It is composed of 6 prismatic joints and 12 motorized passive revolute joints. In this structure the joints were removed.

Each kinematics chain to a structure consisting of 3 bodies with 3 joints [11]. The situation of the first revolute axis of each kinematics chain is shown in Figure 2.

1.1. Geometrical parameters of the kinematics chain i (i=1…6)

Table 1. : Setting geometric chain *i*.

J_i	$a(ji)$	μ_{ji}	σ_{ji}	γ_{ji}	b_{ji}	α_{ji}	d_{ji}	θ_{ji}	r_{ji}
$1i$	0	0	0	γ_{1i}	b_{1i}	$-\pi/2$	d_{1i}	q_{1i}	0
$2i$	$1i$	0	0	0	0	$\pi/2$	0	q_{2i}	0
$3i$	$2i$	1	1	0	0	$\pi/2$	0	0	q_{3i}

for i=1…6; $\gamma_{11} = b_{11} = \gamma_{12} = b_{12} = b_{16} = 0$

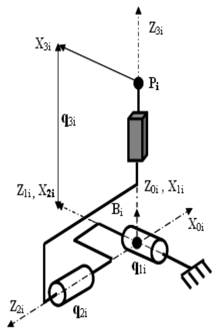

Fig.2: Location frame joints of each kinematics chain.

The geometric parameters of the structure are given in Table 1, we note:

- $\mu(j)$ and $\sigma(j)$ that describe the type of joint; $\mu(j) = 1$ if the joint *j* is motorized and $\mu(j) = 0$ if it is passive.

$\sigma(j) = 1$ if the joint is prismatic and $\sigma(j) = 0$ if it is revolute. (γ_j, b_j, a_j, d_j, θ_j, r_j)

The parameters (γ_j, b_j, a_j, d_j, θ_j, r_j) are used to set the frame *Rj* in reference of its antecedent *Ri*.

The transformation matrix (*T*) consists of these parameters is given by:

$$ {}^{i}T_j = \begin{vmatrix} {}^{i}\mathrm{A}_j & {}^{i}P_j \\ 0_{(1*3)} & 1 \end{vmatrix} $$

Where to:

${}^{i}\mathrm{A}_j$: is the matrix (3 * 3) which defines the direction of the coordinate *Rj* in the coordinate *Ri* .

$$ {}^{i}A_j = \begin{vmatrix} {}^{i}s_j & {}^{i}n_j & {}^{i}a_j \end{vmatrix} $$

${}^{i}P_j$: is the position vector (3 * 1) which defines the origin of the coordinate *Rj* in the coordinate *Ri*.

2. GEOMETRICAL MODEL OF THE ROBOT.

2.1. Direct geometrical Model of chain i

The direct geometric model of a leg *i* robot expresses the operational coordinates (X) of point P_i in frame R_0 according to the joint variables ($q_i = [q_{1i}\, q_{2i}\, q_{3i}]^T$ for i = 1 to 6).

$$ X = f(q) \tag{1} $$

We use the transformation matrices to define the coordinate R_{3i} origin P_i, which is the frame of the chain i in the reference base of the robot R_0:

$$ {}^{0}T_{3i} = {}^{0}T_{1i}\, {}^{1i}T_{2i}\, {}^{2i}T_{3i} \tag{2} $$

The geometric model solution of the parallel robot is not unique since for a given joint configuration variables, the platform can take several different situations.

2.2. Inverse geometric model

The Inverse geometric model is to calculate the joint coordinates (q_i) corresponding to a given situation of the terminal body (platform) based on operational details.

$$ q_i = f(X) \tag{3} $$

This model is easily calculated using some basic elementary geometric relationships (Fig. 3).

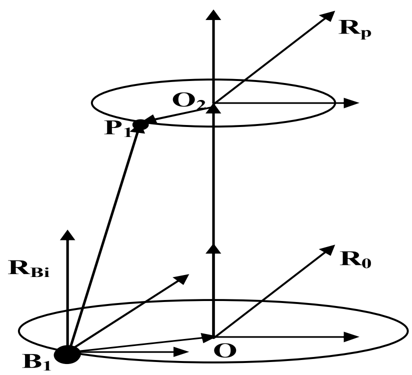

Fig.3: Representation vector $\overrightarrow{BP_{(i)}}$

$$ \overrightarrow{BP_{(i)}} = R\overrightarrow{O_2P_{(i)}} + \overrightarrow{OO_2} + \overrightarrow{B_{(i)}O} \tag{4} $$

R: matrix orientation of the platform.

$$ R = rot(Z,\phi) * rot(Y,\theta) * rot(X,\psi) \tag{5} $$

Let ${}^{Bi}P_{xi}$, ${}^{Bi}P_{yi}$, ${}^{Bi}P_{zi}$ coordinates vectors $BP(i)$ from a frame R_{Bi}. The prismatic variables q_{3i} are obtained by the following equation:

$$q_{3i} = \sqrt{{}^{Bi}P_{xi}^2 + {}^{Bi}P_{yi}^2 + {}^{Bi}P_{zi}^2} \quad (6)$$

The variables relating to passive joints q_{1i} and q_{2i} are given by equations (7), obtained using the method of Paul [1]:

$$-{}^{Bi}P_{xi} S_{1i} + {}^{Bi}P_{yi} C_{1i} = 0 \quad (7)$$

Note: $Sqj_i = Sj_i$ et $Cqj_i = Cj_i$

$$\begin{cases} {}^{Bi}P_{xi}C_{1i}C_{2i} + {}^{Bi}P_{yi}S_{1i}C_{2i} + {}^{Bi}P_{zi}S_{2i} = 0 & (8) \\ -{}^{Bi}P_{xi}C_{1i}S_{2i} - {}^{Bi}P_{yi}S_{1i}S_{2i} + {}^{Bi}P_{zi}C_{2i} = 0 & (9) \end{cases}$$

$$\begin{cases} q_{1i} = \operatorname{atan} 2\left({}^{Bi}P_{yi}, {}^{Bi}P_{xi}\right) & (10) \\ \quad \text{ou} & \\ q'_{1i} = q_{1i} + \pi & (11) \end{cases}$$

and:

$$q_{2i} = \operatorname{atan2}\left(\frac{-q_{3i}\left({}^{Bi}P_{1i}C_{1i} + {}^{Bi}P_{1i}S_{1i}\right)}{Det}, \frac{-q_{3i}\,{}^{Bi}P_{3i}}{Det}\right) \quad (12)$$

with:

$$Det = {}^{Bi}P_{3i}^2 + \left({}^{Bi}P_{1i}C_{1i} + {}^{Bi}P_{2i}S_{1i}\right)^2 \neq 0$$

The condition $Det \neq 0$ is verified as ${}^{Bi}P_{zi}$ is strictly positive and not zero (practically zero means that the chain is collinear with the plane of the base which is impossible), the variable is physically q_{1i} joint between zero and π.

3. KINEMATIC MODEL OF THE ROBOT.

3.1. Direct Kinematics model of the chain i:

The direct kinematics model of the chain i gives the linear velocity of point P as a function of velocity joints of the chain i $\left(\dot{q}_{1i}, \dot{q}_{2i}, \dot{q}_{3i}\right)$ for i = 1..6. This model is identical to the direct kinematics model of a serial structure with three joints (RRP).

$${}^{0}V_{pi} = {}^{0}J_{3i}\dot{q}_i \quad (13)$$

Vpi: linear velocity of point Pi;
${}^{0}J_{3i}$: Jacobian matrix of the kinematics chain i.

3.2. Inverse Kinematics model:

The inverse kinematics model allows expressing the linear speed of the motorized joint variables q as a function of kinematics torsor mobile platform is given by the following equation:

$$\dot{q} = {}^{0}J_p^{-1}\begin{pmatrix} {}^{0}V_p \\ {}^{0}\omega_p \end{pmatrix} \quad (14)$$

${}^{0}V_p$, ${}^{0}\omega_p$: translational speed and angular velocity of the mobile platform;
${}^{\circ}J_p^{-1}$: inverse Jacobian matrix (6 * 6) of the robot.

3.3. Calculating the inverse Jacobian J^{1}_{p}:

The calculation of the inverse of the Jacobian matrix platform is based on determining the speed q_{3i} with the projection of the speed of point Pi on the axis z_{3i}.
We have for a prismatic joint (σ_k=1):

$$\dot{q}_{3i} = {}^{0}\mathrm{a}_{3i}^{T}\,{}^{0}V_{pi} \quad (15)$$

${}^{0}V_{pi}$ linear speed of point P_i, is computed according to V_p and ω_p by modeling of kinematics chains. ${}^{0}\mathrm{a}_{3i}$: unit vector carried by the axis of articulation z_{3i} prismatic.

$${}^{0}V_{pi} = {}^{0}V_p + {}^{0}\omega_p \times {}^{0}L_i \quad (16)$$

${}^{0}L_i$: vector representing the components of P_1P_i vector expressed in the frame R_0.
It comes as:

$$\dot{q}_{3i} = {}^{0}\mathrm{a}_{3i}^{T}\,{}^{0}V_P + {}^{0}\mathrm{a}_{3i}^{T}\left({}^{0}\omega_p \times {}^{0}L_i\right) \quad (17)$$

and then:

$$\dot{q}_{3i} = {}^{0}\mathrm{a}_{3i}^{T}\,{}^{0}V_P + \left({}^{0}L_i\,{}^{0}\mathrm{a}_{3i}\right)^T {}^{0}\omega_p \quad (18)$$

$$\dot{q}_{3i} = {}^{0}\mathrm{a}_{3i}^{T}\left(\left({}^{0}\hat{L}_i\mathrm{a}_{3i}\right)^T\begin{pmatrix} {}^{0}V_p \\ {}^{0}\omega_p \end{pmatrix}\right) \quad (19)$$

$${}^{0}J_p^{-1} = \begin{pmatrix} {}^{0}\mathrm{a}_{31}^{T} & \left({}^{0}\hat{L}_1\,{}^{0}\mathrm{a}_{31}\right)^T \\ . & . \\ . & . \\ . & . \\ . & . \\ {}^{0}\mathrm{a}_{36}^{T} & \left({}^{0}\hat{L}_6\,{}^{0}\mathrm{a}_{36}\right)^T \end{pmatrix} \quad (20)$$

In section five, a MATLAB simulation is required to view and verify the results obtained from the Jacobian matrix inverse ${}^{0}J_p^{-1}$. (Fig.4)

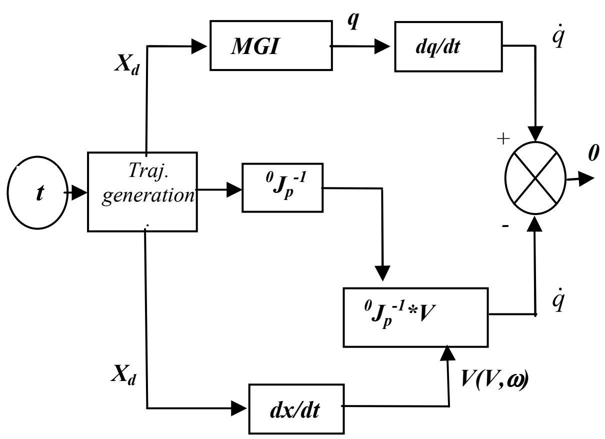

Fig.4: Global Scheme used to validate the analytical kinematics model.

4. INVERSE DYNAMIC MODEL

The inverse dynamic model is the relationship between the forces of the motorized joint and positions, velocities, accelerations of the platform. The inverse dynamic model is intended to control robots.

$$\Gamma = f\left({}^0T_p, {}^0V_p, {}^0\dot{V}_p\right) \tag{21}$$

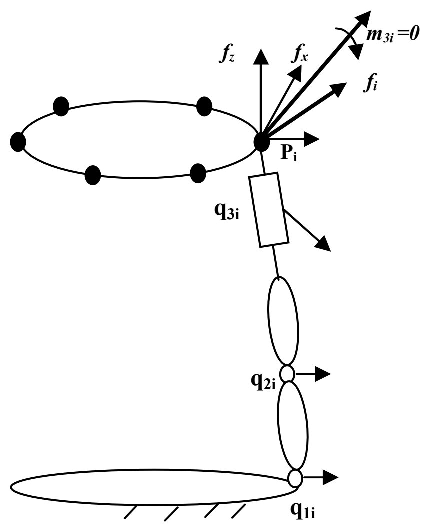

Fig. 5: Forces and moments applied on the platform by the forces of reaction chains.

For the dynamic model we decompose the system into two subsystems: the platform that is linked to the kinematics chain of joints, the torque reaction transmitted to the chain at a platform is zero, the effect of chain i kinematics on the platform is represented by reaction forces f_i (i = 1 to 6).
The tree structure (the second subsystem) consists of the base and legs with two pairs of revolute joints (Γ_{1i} , Γ_{2i}) are zero.

In this problem the unknowns are the 18 components of reaction forces ${}^0f_i$ = $[{}^0f_{xi}\ {}^0f_{yi}\ {}^0f_{zi}]^T$ and the 6 forces motorized joints Γ_{3i} (i =1 à 6). Each dynamic model of a kinematics chain is composed of three equations, giving a total of 18 equations. The Newton-Euler equations of the platform gives 6 equations, we obtain a system of 24 equations in 24 unknowns.
This system will be solved sequentially as follows:
- calculation of ${}^0f_i$ based Γ_{3i} using the dynamic model of the kinematic chain i
- Γ_{3i} forces will be obtained from Newton-Euler equations of the platform.

4.1 Calculating the reaction force ${}^0f_i$

The general form of the dynamic model of the kinematics chain i, is written:

$$\Gamma_i = \mathrm{H}_i\left(q_i, \dot{q}_i, \ddot{q}_i\right) + {}^0J_{3i}\,{}^0f_i^T \tag{22}$$

With:
H_i: vector (3 * 3) containing the inertial forces, Coriolis, centrifugal and gravity.
Γ_i : is composed of forces / torques of the joints of the chains where Γ_{1i} and Γ_{2i} are zero;

$$\Gamma = \begin{bmatrix}\Gamma_{1i} & \Gamma_{2i} & \Gamma_{3i}\end{bmatrix}^T = \begin{bmatrix}0 & 0 & \Gamma_{3i}\end{bmatrix}^T \tag{23}$$

From equation (22) of the general form, the reaction force of the kinematics chain i can be written:

$${}^0f_i = -{}^0J_{3i}^{-T}\mathrm{H}_i\left(q_i, \dot{q}_i, \ddot{q}_i\right) + {}^0J_{3i}^{-T}\Gamma_i \tag{24}$$

4.2 Dynamic of platform

The Newton Euler equations around the origin of the platform are written:

$${}^0F_p = {}^0\mathrm{II}_p\,{}^0\dot{V}_p + \begin{pmatrix}{}^0\omega_p \times \left({}^0\omega_p \times {}^0MS_p\right) \\ {}^0\omega_p \times \left({}^0I_p \times {}^0\omega_p\right)\end{pmatrix} - \begin{pmatrix}M_pI_3 \\ {}^0M\hat{S}_p\end{pmatrix}{}^0g \tag{25}$$

With :
${}^0F_p$: external forces and moments applied on the platform.

${}^0f_i$ = $[{}^0f_{xi}\ ,{}^0f_{yi}\ ,{}^0f_{zi}]^T$ reaction force of the chain i and 6 forces motorized joints Γ_{3i} (i= 1à 6).
${}^0\mathrm{II}_p$: space inertia matrix (6 * 6) of the platform, we have :

$${}^0\mathrm{II}_p = \begin{pmatrix}M_pI_3 & -{}^0M\hat{S}_p \\ {}^0M\hat{S}_p & {}^0I_p\end{pmatrix} \tag{26}$$

${}^0I_p$: inertia tensor (3 * 3) of the platform expressed in the frame R_0, which is expressed by:

$${}^0I_p = {}^0A_p\,{}^pI_p\,{}^0A_p^T \tag{27}$$

${}^{p}I_{p}$: inertia tensor (3 * 3) of the platform expressed in the coordinate R_p and ${}^{0}A_p$ matrix orientation that expresses the frame R_p in the frame R_0.

$$ {}^{p}I_{p} = \begin{pmatrix} XX_p & XY_p & XZ_p \\ XY_p & YY_p & YZ_p \\ XZ_p & YZ_p & ZZ_p \end{pmatrix} \quad (28) $$

${}^{0}MS_p$: first moment of inertia of the platform around the origin of the frame R_p.

$$ {}^{p}MS_{p} = \begin{bmatrix} MX_p & MY_p & MZ_p \end{bmatrix}^T \quad (29) $$

$$ {}^{p}MS_{p} = {}^{0}A_p \, {}^{p}MS_p \quad (30) $$

I_3: Identity matrix (3*3) ; M_p: mass of the platform ; ${}^{0}g$: acceleration of gravity.

4.3. Relationship between ${}^{\circ}F_p$ and ${}^{\circ}f_i$:

The forces and moments applied on the origin of the platform by the reaction forces of kinematics chains are given by the following equation:

$$ {}^{0}F_{p} = \sum_{i=1}^{6} \begin{pmatrix} I_3 \\ {}^{0}\hat{L}_i \end{pmatrix} {}^{0}f_i \quad (31) $$

Equation (24) allows to write:

$$ {}^{0}F_{p} = \sum_{i=1}^{6} \left(\begin{pmatrix} I_3 \\ {}^{0}\hat{L}_i \end{pmatrix} \left(-{}^{0}J_{3i}^{-T} \mathrm{H}_i(q_i, \dot{q}_i, \ddot{q}_i) + {}^{0}J_{3i}^{-T} \Gamma_i \right) \right) \quad (32) $$

Let H_{xi} expressing the vector $\mathrm{H}_i(q_i, \dot{q}_i, \ddot{q}_i)$ in cartesian space to the point P_i :

$$ \mathrm{H}_{xi} = {}^{0}J_{3i}^{-T} \mathrm{H}_i(q_i, \dot{q}_i, \ddot{q}_i) \quad (33) $$

With:

$$ {}^{0}J_{3i}{}^{-T} \Gamma_i = {}^{0}A_{3i} \, {}^{3i}J_{3i} \Gamma_i \quad (34) $$

$$ {}^{0}J_{3i}^{-T} \Gamma_i = {}^{0}A_{3i} \begin{vmatrix} 0 \\ 0 \\ \Gamma_{3i} \end{vmatrix} = {}^{0}a_{3i} \Gamma_{3i} \quad (35) $$

With: $${}^{3i}J_{3i}^{-T} = \begin{bmatrix} 0 & 1/q_{3i}\sin(q_{2i}) & 0 \\ 1/q_{3i} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equation (34) can be written:

$$ \sum_{i=1}^{6} \left(\begin{pmatrix} I_3 \\ {}^{0}\hat{L}_i \end{pmatrix} {}^{0}a_{3i} \Gamma_{3i} \right) = {}^{0}F_p + \sum_{i=1}^{6} \left(\begin{pmatrix} I_3 \\ {}^{0}\hat{L}_i \end{pmatrix} \mathrm{H}_{xi}(q_i, \dot{q}_i, \ddot{q}_i) \right) \quad (36) $$

Using the equation of the Jacobian inverse, we can deduce that:

$$ \sum_{i=1}^{6} \left(\begin{pmatrix} I_3 \\ {}^{0}\hat{L}_i \end{pmatrix} {}^{0}a_{3i} \Gamma_{3i} \right) = {}^{0}J_p^{-T} \Gamma \quad (37) $$

With:

$$ \Gamma = [\Gamma_{31} \ldots\ldots \Gamma_{36}] \quad (38) $$

Substituting equation (38) in equation (30), we obtain:

$$ \Gamma = {}^{0}J_p{}^{T} \left({}^{0}F_p + {}^{0}F_{chaine} \right) \quad (39) $$

With:

$$ F_{chaine} = \sum_{i=1}^{6} \left(\begin{pmatrix} I_3 \\ {}^{0}\hat{L}_i \end{pmatrix} \mathrm{H}_{xi}(q_i, \dot{q}_i, \ddot{q}_i) \right) \quad (40) $$

Equation (39) represents the inverse dynamics model of the robot without friction and inertia of the actuators. To complete this model, we introduce the friction and inertia of the actuators. The expression (39) becomes:

$$ \Gamma = \mathrm{diag}(\dot{q}_a) F_s + \mathrm{diag}(\mathrm{sign}(\dot{q}_a)) F_v + \mathrm{diag}(\ddot{q}_a) M_a + {}^{0}J_p^T F_{robot} \quad (41) $$

Where:

diag(.) : represents a matrix (6 * 6) with the terms in parentheses on the diagonal non-zero and other zero;

$$ diag(\dot{q}_a) = \begin{bmatrix} \dot{q}_{31} & 0 & 0 & 0 & 0 & 0 \\ 0 & \dot{q}_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot{q}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dot{q}_{34} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dot{q}_{35} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dot{q}_{36} \end{bmatrix} \quad (42) $$

sign(.) : represents the sign function;
F_s: vector (6 * 1) composed of dry friction parameters of the actuators;
F_v :vector (6*1)composed of viscous friction parameters of the actuators.
H_i (q_i, $\dot{q}_i$, $\ddot{q}_i$) representing the inverse dynamics model of the chain i is calculated by Newton's - Euler method [8].
Equation (42) represents the inverse dynamics model. For the calculation, in addition to the calculation of Jacobian matrices, we must determine $\mathrm{H}_i(q_i, \dot{q}_i, \ddot{q}_i)$ which is simply the inverse dynamic model of the chain i expressed in cartesian space.

It summarizes the steps of calculating the inverse dynamics model as follows:

- Resolution of geometric and kinematics direct models ;
- Calculation of dynamic model of each kinematics chain : $\mathrm{H}_i(q_i, \dot{q}_i, \ddot{q}_i)$;
- Calculation of the Cartesian dynamic model of each kinematics chain : H_{xi} ;
- Calculation of forces and moments ${}^0\mathrm{F}_{\mathrm{chaine}}$ that match all H_{xi} ;
- Determine the forces and / or moments required to move the platform, ${}^0\mathrm{F}_{\mathrm{p}}$;
- Calculation of forces and / or desired moments: $\Gamma = {}^0J_p^T\,({}^0\mathrm{F}_{\mathrm{p}} + {}^0\mathrm{F}_{\mathrm{chaine}})$.

Physical interpretation

We note from equations (38) (39) and (40), the effect of kinematics chains on the platform is equivalent to a force equal to $-\mathrm{H}_{xi}(q_i, \dot{q}_i, \ddot{q}_i)$ at each point P_i. The corresponding total force is equal to $\sum \mathrm{H}_{xi}(q_i, \dot{q}_i, \ddot{q}_i)$.

The parallel robot we just studied can be represented by a single body with the same parameters as the inertial platform on which is applied to each point P_i a force $-H_{xi}$.

5. SIMULATION AND RESULTS

The overall pattern of the simulation (figure 4) reflects the summary of the method developed for the kinematics analysis. Indeed, the inverse jacobian matrix of the robot (equation 20) allows deducing the velocities of the segments (cylinders). By direct derivation of active joint variables (results of inverse geometrical Model) yields the same vector velocity $\dot{q}_{3i}$. The estimate of the error term is used to validate the results of the analytical method.

To this end, using a trajectory generator whose parameters reflect the position and orientation (rotations about the base z-axis φ, y-axis θ and x-axis Ψ) of the platform at any time *t* (desired situation). The numerical values of parameters used in the simulation reveal two important factors:

- General form and dimensions adopted for the structure of the robot. A conceptual study of a robot type Gough-Stewart CAD (Solid Works) at identified all the dimensional parameters and inertial i.e. mass, center of gravity, inertia, and dimensions.
- Singularities. The values given to the angles (φ, θ, Ψ) must lead to positions available (joint limits).

In order to validate simulation models proposed considering the following physical data:

a. Movement of the platform.

$t = 0 : 10$ sec.

Position and orientation of the origin of the frame R_p

$$P_{x0} = 10 * t \qquad \varphi = 0.07 * \mathrm{t}$$
$$P_{y0} = 0 \qquad \theta = 0.04 * \mathrm{t}$$
$$P_{z0} = 800 + 50 * t \qquad \psi = 0$$

b. Masses.

Platform M= 24.45 kg
Kinematics chains *m =[0.11818 ; 7.27885 ; 3.15325]*

c. Moments of the 2nd order of chains [kg.m^2].

K (:,:,1)=[18822.31 0 0 ; 0 11453.35 0 ; 0 0 11343.03]
K(:,:,2) =[95191796 2745801 31799761.94; 2745801 106222355.34 13430543; 31799761.94 13430543 34166734]
K(:,:,3)=[136247234.57 0 0 ; 0 136234387.21 0 ; 0 0 693106.67].

d. Moments of inertia of the platform around the origin of the coordinate R_p [kg.m^2].

Ixx =6246766024.15; *Iyy*=612114662.55; *Izz*=1222025614.27;

For a simple configuration chosen in advance, we can see the corresponding movement of the platform (Fig.6).

It is shown in Figures 7, 8, 9 and 10 the change of variables and joint velocities q_{3i}, $\dot{q}_{3i}$, and the different speeds of the platform *Vp*.

Figure 11 shown for the same physical data, the variation of active forces in joints obtained from the equations of Newton - Euler governing the dynamics of the platform.

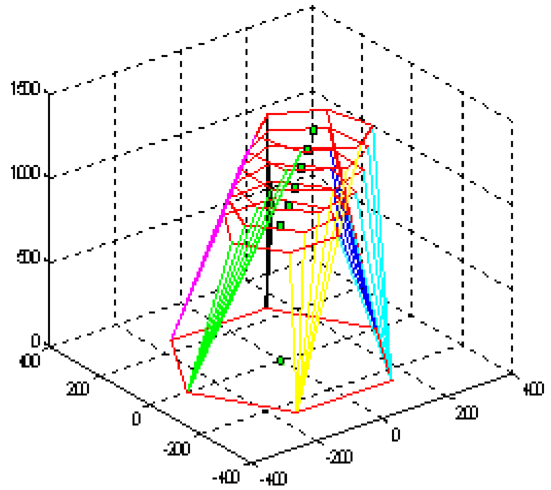

Fig.6: Matlab simulation of a space trajectory's robot.

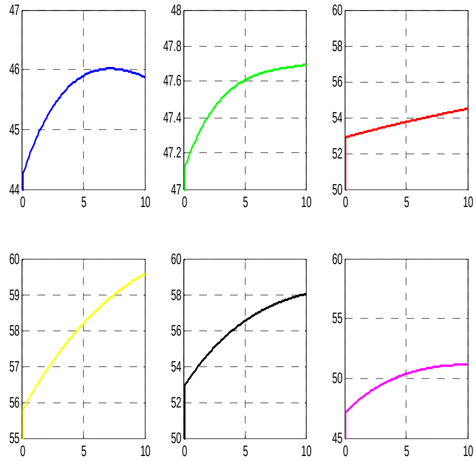

Fig. 9: Speed actuators "$\dot{q}_{3i}$" [mm/sec.].

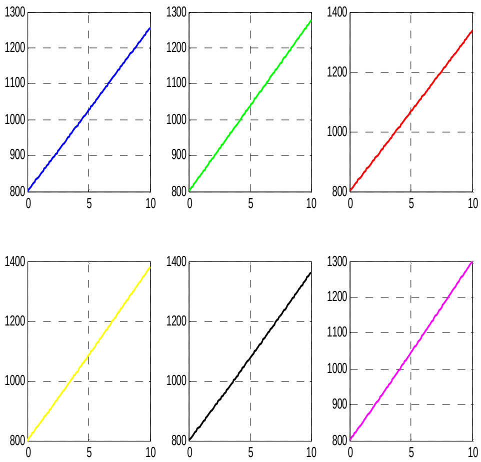

Fig. 7: Displacement of active joints q_{3i} [mm].

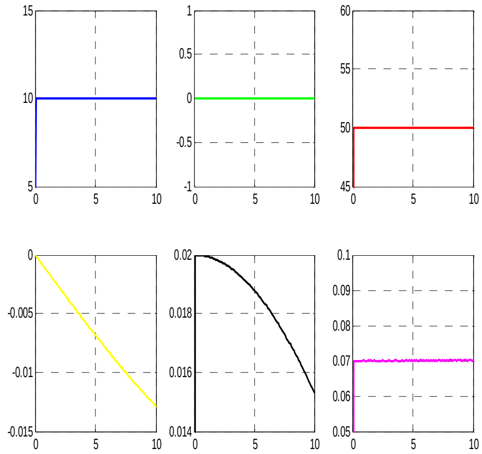

Fig. 10: Linear and angular velocities of the Platform" V_{0p}" [mm/sec.] and [rd/sec.].

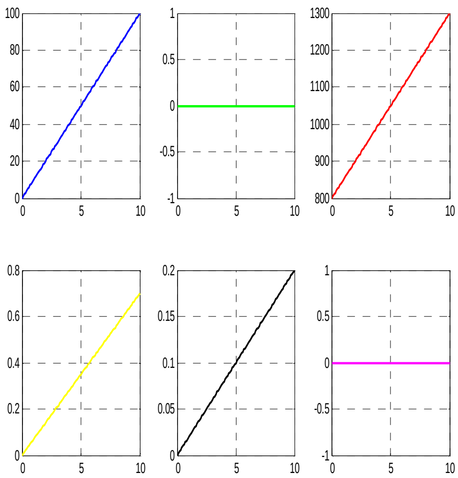

Fig.8: Displacement and orientation of the platform in [mm] and [rd].

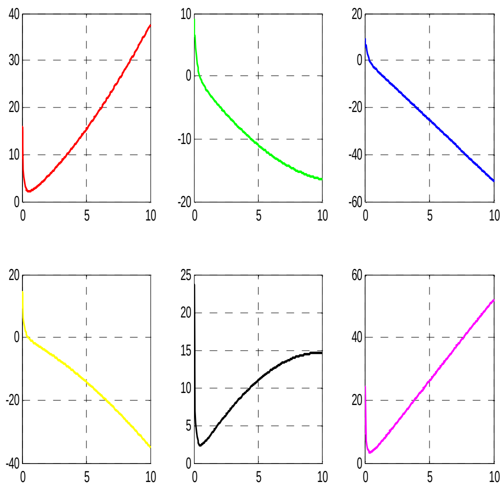

Fig. 11: Change Γ_{3i} forces at the actuators [N].

CONCLUSION:

In this paper, a dynamic model of the parallel robot with six degrees of freedom has been made. A description based on the decomposition of the robot into two parts: the mobile platform and chains (legs) attached to the base forming a tree structure, to permit application of the methods known and used for robot series, branching or closed loop.
Geometric modeling and kinematics of the robot was developed on the basis of previous work [7], [10] and [14]. The proposed models take into account the dynamics of the robot type Gough - Stewart. The calculation of these models is facilitated by being able to apply the techniques developed for serial robots. The value of modeling dynamic developed is that it allows to deduce directly a physical interpretation of the model. Indeed the one body equivalent to the robot has the same parameters as the inertial platform which is applied a force vector whose points of application are connections to the chains. There is therefore a compact algorithm capable of solving the dynamic a parallel robot in view of its adaptation to dynamic control laws.
It plans to use and extend this approach to what we call the hybrid structure [10], serial – parallel robots.

BIBLIOGRAPHY

[1] Wissama Khalil, "modélisation identification et commande des robots"

[2] Merlet .J-P.," parallel robots ", Kluwer Academic Publ., Dortdrech, the Neterland, 2000

[3] Ait Ahmed M., "Contribution à la modélisation géométrique et dynamique des robots parallèles", Thèse de doctorat, LAAS, Toulouse, 1993.

[4] Tsai L-W., "Solving the inverse dynamic of Gough – Stewart manipulator by the principle of virtual work" , J. of Mechanical design, Vol. 122, P. 3-9, mars 2000.

[5] Liu M. J.,Li C-X, Li C-N "dynamic analysis of the Gough – Stewart platform manipulator" ,IEEE Transaction on robotics and Automation, Vol. 16(1), P. 94-48, Février 2000.

[6] Borojéni D.," Modélisation cinématique et dynamique des système polyarticulés à chaînes ouvertes ou fermées". Cas des robots parallèles, Thèse de doctorat, Université Paris XII- Val de Marne, 2006.

[7] Guegan S.,"Contribution à la modélisation et identification dynamique des robots parallèles", Thèse de doctorat, ECN, Nantes, 2003.

[8] Gautier M., " Contribution à la modélisation et à l'identification des robots", Thèse d'état, Université de Nantes, ENSM, Mai 1990.

[9] Khalil W., Kleinfinger J.-F., »A new geometric notation for open and closed-loop robots», Proc.IEEE conf .on Robotics and Automation, San Fransisco, avril 1986, p1174-1180.

[10] Ourda Ibrahim, "Contribution à la modélisation dynamique des robots parallèles et des robots hybrides", Thèse de doctorat, année 2006

[11] Pierre Renaud, "Kinematic and dynamic identication of parallel mechanisms; Article, Control engeneering practice 14 (2006) 1099-1109

[12] Oscar Andrés V. A., "Contribution à l'identification et à la commande des robots parallèles", Thèse de doctorat, Univ. Monpelier II, novembre 2004.

[13] Guegan S., Khalil W., "Modélisation dynamique d'un robot parallèle à trois degrés de liberté : l'Orthoglide", Article année 2004, IRCCyN.

[14] Ibrahim O., Khalil W., Guegan S., "Dynamic modeling of some parallel robots", 35th International Symposium on robotics, Villepinte, France, 23-26 Mars, 2004.

[15] C. Mahfoudi , K. Djouani, M. Bouaziz and S. Rechak, "General method for dynamic modeling and control of an hexapod robot including optimal force distribution", J. WSEAS, Transaction on signal processing, Vol. 2, P. 323-330, 2006.

[16] C. Mahfoudi , K. Djouani, S. Rechak and M. Bouaziz, "Optimal force distribution for the legs of an hexapod robot", IEEE Conference on control Application CCA, Istanbul, Vol. 1, P. 65-670, 2003.

[17] Stefan Staicu, "Modèle matriciel en dynamique du robot parallèle Stewart - Gough", Rev. Roum., sci. Tech. -Méc. -Appli., Tome 54, No 3, Bucarest : 2009.