Optimal grasping force distribution for three robots holding a rigid object

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Abstract

In this paper we present a methodology for optimal force distribution calculation for the multiple manipulators system grasping an object.We consider for this case three robots holding a common rigid object with three contact points , where the object can translate and rotate in any directions. This approach is used in the case of real time dynamics object force control. The force distribution problem is formulated in terms of a nonlinear programming problem under equality and inequality constraints. Then, according to [1],[2],[3] and [4] the friction constraints are transformed from non linear inequalities into a combination of linear equalities and linear inequalities. The original non linear constrained programming problem is then transformed into a quadratic optimization problem. Some simulation results are given to show the efficiency and accuracy of the proposed methodology and perspectives on multiple manipulators system grasping an object control are discussed.
Keywords : Multiple Manipulators System, Grasping, Optimal Force Distribution, Quadratic Programming, Friction Constraints.

1 Introduction

Cooperative system is generally understood as several coordinated robots simultaneously performing a given task. The main objective of a multi-robots cooperative system in robotics is to manipulate an object. Manipulation is performed with the aim of, for example , changing the space position of an object,grasping an object in contact with environment , gripping,lifting, lowering, releasing, withdrawing...

The object force control need calculation of real-time force distribution on the robot's efectors. Due to the existence of more than three actuated joints in each robot , the manipulators system has redundant actuation leading to more active joints than the object degree-of-freedom (6 dof). Thus, when formulating the force distribution problem, we find fewer force moment balancing equations than unknown variables. So, the solution of these equations is not unique. Moreover, some physical constraints, that concern the contact nature, friction, ...etc , must be taken into account in the calculation of force distribution. In addition, joints torque saturation must also be considered. Several approaches in the literature have been proposed to address robot coordination problem in [5],[6] Contact and friction constraints for grasp conditions are considered and a optimization algorithm is developed by minimizing the total energy E consumed by the actuators of the planar dual-arm manipulators system.A new approach for computing force-closure grasps of two-dimensional and three-dimensional objects was proposed in [7]-[8]and [9] then a new necessary and sufficient condition for n-finger grasps to achieve force-closure property are developed.in [10] The analysis of grasping and manipulation of deformable objects by a three finger robot hand has been carried out.Thus the Force Distribution Problem (FDP) can be formulated as a nonlinear constrained programming problem under nonlinear equality and inequality constraints.

- Linear-Programming (LP) Method [3],[11]
- Compact-Dual LP (CDLP) Method [12],[13]
- Quadratic Programming (QP) Method [14], [15]
- Analytical Method [16],[17],[18],[19]

A comparative study for the cited methods can be found in [20]. Some researchers proposed the optimal force distribution scheme of multiple cooperating robots by combining the Dual Method with the

QP [21]. In FDP solving, according to some criteria, physical aspects of the multiple cooperating robots have to be considered. In this paper we propose an approach to solve the problem of real-time force distribution for a multiple manipulators system, based on the approach proposed in [22],[23] for an hexapod robot.

This approach consists of the combination of the QP Method with the reduction technique of problem size. The main idea concerns the transformation of the original nonlinear constrained problem into a linear one, by reducing the problem size and transforming the nonlinear constraints into a linear ones, respecting some physical considerations.

The rest of the paper is organized as follows. The direct and inverse geometrical models of the robot manipulator are presented in section 2. Section 3 concerns the force distribution problem. Problem reduction and optimal solution are presented in section 4. Before presenting some remarks and perspectives, a Matlab simulation results of two cooperating manipulators is presented in section 5 to show the efficiency of this approach.

2 Geometrical Modelling

Before presenting the direct and inverse geometrical models, let us consider the robot architecture. As all the robot are identical, only one robot modelling is considred, the robot j architecture is given in figure (2). Every robot "j" j=1,...,n is grasping the object, located at a_j distance from the center of gravity of the object. The angle ϕ_j represents the orientation of the coordinate frame $(x_{1,j}, y_{1,j}, z_{1,j})$ fixed at the first articulation of the robot and the world ground coordinate frame (X, Y, Z). Multiple Manipulators System is considerated as an arborescent robot comporting some closed loops. So to study this kind of robots we use the method defined by Khalil and Kleinfinger [24].

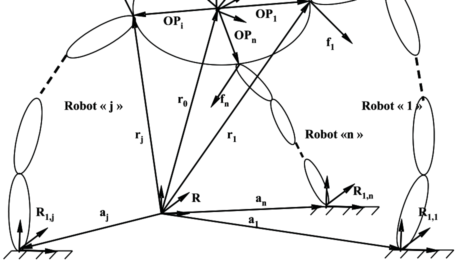

Figure 1: **Geometrical parameters of multiple manipulators system**

The transformation matrix from ith joint's attached coordinate frame to the (i-1)th joint's attached coordinate frame is given by figure (3):

$$^{i-1}\boldsymbol{T}_i = R(Z,\gamma)T(Z,b)R(X,\alpha)T(X,d)R(Z,\theta)T(Z,r) \quad (1)$$

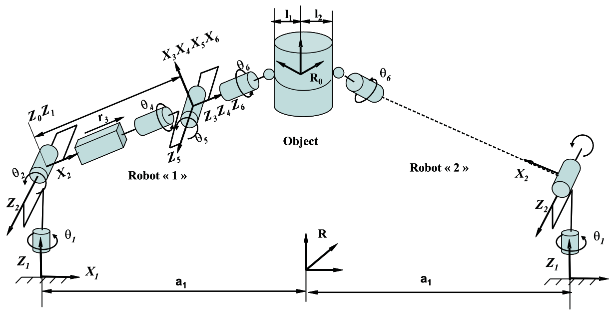

Figure 2: **Exemple of multiple manipulators system**

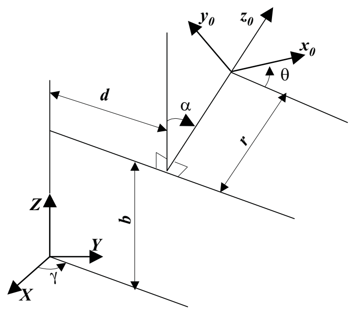

Figure 3: **geometrical model**

The table (1) describes the transformation from the world ground coordinate frame (X, Y, Z) to the coordinate frame at the contact point "6" of each robot.

frame	α	d	θ	r	b	γ
object	α	d	θ	r	h	β
Joint"1"	0	OP_1	$\theta_{1,1}$	0	0	0
Joint"j"	0	OP_j	$\theta_{1,j}$	0	0	0
Joint"n"	0	OP_n	$\theta_{1,n}$	0	0	0

Table 1: **Geometrical parameters of the object**

The transformation providing the exact position of the contact point"6" of any robot in the absolute coordinate frame fixed at the ground is given by :

$$^R\boldsymbol{T}_6 = ^R\boldsymbol{T}_0^0\boldsymbol{T}_1^1\boldsymbol{T}_2^2\boldsymbol{T}_3^3\boldsymbol{T}_4^4\boldsymbol{T}_5^5\boldsymbol{T}_6 \quad (2)$$

When the position and the orientation of the last coordinate frame fixed to the end of each robot"j" are known ,We apply the method proposed by Paul [25]. It provides the values of the joints coordinates $\theta_{i,j}$ $(i = 1, 2, 3, 4, 5, 6)$ $(j = 1, .., n)$ as follow:

frame	α	d	θ	r	b	γ
Joint"1"	0	a_j	θ_1	0	0	ϕ_j
Joint"2"	0	0	$\theta_2+\pi/2$	0	0	0
Joint"3"	$\pi/2$	0	0	r_3	0	0
Joint"4"	0	0	θ_4	r_4	0	0
Joint"5"	$-\pi/2$	0	θ_5	0	0	0
Joint"6"	$\pi/2$	0	θ_6	0	0	0

Table 2: **Geometrical parameters of the jth robot**

$$\left\{\begin{array}{l}\theta_{1,j}=\arctan(Py/(-Px+d_1))\\ \theta_{2,j}=\arctan(Pz-r_1)/(C1(Px-d_1)+S1Py))\\ r_{3,j}=C2(C1(Px-d_1)+S1Py)+S2(Pz-r_1)-\\ \quad -R4-l_2\\ \theta_{4,j}=\arctan((S1ax-C1ay)/(S2(C1ax+S1ay)-\\ \quad -C2az))\\ \theta_{5,j}=\arctan((C4S2(C1ax-S1ay)-C4C2az\\ \quad +S4(C1ay-C1sy))/(C2(C1ax+S1ay)+\\ \quad +S2az))\\ \theta_{6,j}=\arctan((-S4(-S2(C1ax-S1ay)+C2sz+\\ \quad C4S1sx-C1sy))/(C5(-C4S2C1sx-S2S1sy+\\ \quad +C2sz)+S4(S1sx-C1sy)-S5(C1C2sx+\\ \quad +S1C2sy+S2sz))\end{array}\right. \tag{3}$$

Remark : S*=sin(*); C*=cos(*); ($Px=P_{x,j}, Py=P_{y,j}, Pz=P_{z,j}$), are the coordinates of the point "6" of the jth robot expressed in (X,Y,Z).

3 Force Distribution Problem

3.1 problem Formulation

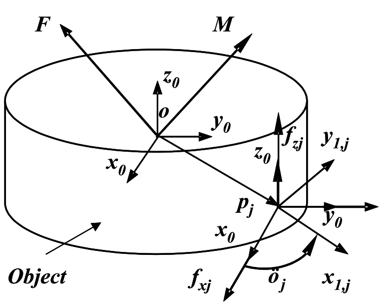

Figure 4: Orientation of coordinate frame

The force system acting on the object is shown in figure (4). For simplicity, only the force components on the contact point are presented here. In the general case, rotational torques at the contact are neglected. Let (x_0, y_0, z_0) be the coordinate frame in which the object is located and $(x_{6,j}, y_{6,j}, z_{6,j})$ denote the coordinate frame fixed at the contact point of the jth robot. The $(x_{6,j}, z_{6,j})$ plane which is assumed to be parallel to the (x_0, y_0, z_0) plane and its z axis is normal to the surface of the object. $\boldsymbol{F}=[F_X F_Y F_Z]^T$ and $\boldsymbol{M}=[M_X M_Y M_Z]^T$ denote respectively the object force vector and moment vector, which results from the gravity and the external force acting on the object [26],[27],[28] and [29]. Define $f_{x,j}, f_{y,j}$, and $f_{z,j}$ as the components of the force acting on the supporting robot "j" in the directions of x_0, $y_0\, and\, z_0$, respectively. The number of supporting robot, n, can vary between 2 and 3 for this studies. The object's quasi-static force/moment equation can be written as

$$\left\{\begin{array}{l}\sum_{j=1}^{n}\boldsymbol{f}_j=\boldsymbol{F}\\ \sum_{j=1}^{n}\boldsymbol{OP}_j\wedge\boldsymbol{f}_j=\boldsymbol{M}\end{array}\right. \tag{4}$$

where $\boldsymbol{OP}_j$ is the position vector joining contact point of the robot "j" and the gravity center of the object. The general matrix form of this equation can be written as :

$$\boldsymbol{A}\,\boldsymbol{G}=\boldsymbol{W} \tag{5}$$

with:

$$\left\{\begin{array}{ll}\boldsymbol{G}=[\boldsymbol{f}_1^T\boldsymbol{f}_2^T\cdots\boldsymbol{f}_n^T]^T & \in\Re^{3n}\\ \boldsymbol{f}_j^T=[f_{x,j}\,f_{y,j}\,f_{z,j}]^T & \in\Re^3\\ \boldsymbol{W}=[\boldsymbol{F}^T\,\boldsymbol{M}^T]^T & \in\Re^6\end{array}\right.$$

$$\boldsymbol{A}=\begin{pmatrix}\boldsymbol{I}_3 & \dots & \dots & \boldsymbol{I}_3\\ \boldsymbol{B}_1 & \dots & \dots & \boldsymbol{B}_n\end{pmatrix}\in\Re^{6\times 3n}$$

$$\boldsymbol{B}_j\equiv\widehat{\boldsymbol{OP}}_j\equiv\begin{pmatrix}0 & -P_{z,j} & P_{y,j}\\ P_{z,j} & 0 & -P_{x,j}\\ -P_{y,j} & P_{x,j} & 0\end{pmatrix}\in\Re^{3\times 3}$$

where $\boldsymbol{I}_3$ is the identity matrix and **G** is the robots force vector, corresponding to three ($\boldsymbol{G}\quad\in\Re^9$) . $\boldsymbol{A}$ is a coefficient matrix which is a function of the positions of the robot supporting, and $\boldsymbol{B}_j$ is a skew symmetric matrix consisting of $(P_{x,j}, P_{y,j}, P_{z,j})$, which is the position coordinate of

contact point of the supporting robot "j" in (x_0, y_0, z_0). $\boldsymbol{W}$ is a total body force/moment vector. It is clear that (5) is an underdetermined system and its solution is not unique. In other words, the robots forces have many solutions according to the equilibrium equation. however, the robot forces must meet the needs for the following physical constraints,

otherwise they become invalid :

[3]Supported object should not slip when the robots move. It results in the following constraint:

$$\sqrt{f_{x,j}^2+f_{y,j}^2}\le\mu\, f_{z,j} \tag{6}$$

where μ is the static coefficient of friction of the surface of the object Since the robots forces are generated from the corresponding actuators of joints, the physical limits of the joint torques must be taken into account. It follows that :

$$-\tau_{jmax}\le\boldsymbol{J}_j^{T\,j}\boldsymbol{A}_{0j}\begin{pmatrix}f_{x,j}\\ f_{y,j}\\ f_{z,j}\end{pmatrix}\le\tau_{jmax} \tag{7}$$

for ($j = 1, ..., n$), where $\boldsymbol{J}_j \in \Re^{3\times6}$ is the Jacobian of the robot "j", $\tau_{jmax} \in \Re^{6\times1}$ is the maximum joint torque vector of the robot "j", and $\boldsymbol{A}_{0j} \in \Re^{3\times3}$ is the orientation matrix of $(x_{6,j}, y_{6,j}, z_{6,j})$ with respect to (x_0, y_0, z_0). In order to have definite contact with the object, there must exist a $f_{z,j}$ such that :

$$f_{z,j} \geq 0 \tag{8}$$

In the following, we propose an approach for problem size reduction, linearisation and solving for the three manipulators case. Clearly, it is difficult to solve such a nonlinear programming problem for real-time multi-robots force distribution with complex constraints.

3.2 Problem Size Reduction

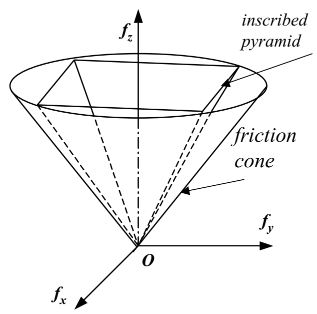

Figure 5: conservative inscribed pyramid

The equation (6) is a formulation of the friction cone figure(5). In order to overcome the non linearities induced by the following equations, most researches substitute this friction cone by the inscribed pyramid [11],[30],[14],Thus, the nonlinear friction constraints are approximately expressed by the following linear inequalities :

$$f_{x,j} \geq \acute{\mu}\, f_{z,j}, \quad f_{y,j} \geq \acute{\mu}\, f_{z,j}, \quad j = 1, ..., n \tag{9}$$

where $\acute{\mu} = \frac{\sqrt{2}\mu}{2}$ is for the inscribed pyramid. Thus, the initial non linear constrained programming problem, substituting the non linear constraint Eq(6) by the linear one of Eq(9), becomes a linear programming problem [11],[12]and [14]. The possibility of slipping can be minimized, by optimizing the ratio of tangential to normal forces at the robot. In [31], the authors have shown that, for multi-robots, all ratios (at the contacts points) are equal to the global ratio. This leads Liu and Wen [17] to find the relationship between the robot forces and transform the initial friction constraints from the nonlinear inequalities into a set of linear equalities. Let us define the global ratio by the ratio of the tangential to normal forces at the object. The advantage of the existing methodes lies in the fact that part of component of the robots forces satisfy the global ratio relation ship and lets the other components satisfy the linear inequality constraints as Eq (9). For example, defining $f_{x,j}$ $(j = 1; ; n)$ and $f_{y,j}$ $(j = 1; ; n)$, for a robot j Chen et al [1], show that :

$$f_{x,j} = k_{xz} f_{z,j} \quad, (i = 1, ..., n) \tag{10}$$

$$f_{y,j} \leq \mu^{\star} f_{z,j} \quad, (i = 1, ..., n) \tag{11}$$

where $k_{xz} = \frac{F_X}{F_Z}$ is the global ratio of forces at the object in direction of x_0 and z_0. $\mu^{\star}$ is the given coefficient for friction constraints. According to Eq(6), we have $\mu^{\star} = \sqrt{\mu^2 - k_{xz}^2}$. Finally, the force distribution problem is transformed into a linear one by replacing Eq (6) with Eqs (10) and (11).

3.3 Problem transformation and Continuous solution

In modelling this systems, we consider that three robots support the object at a time. so $\boldsymbol{G}$ and $\boldsymbol{A}$ become a vector of 9×1 and a matrix of 6×9, respectively. Equation (5) contains nine unknown variables with six equations. By adding the Eq (10) to the Eq (5) we obtain nine equations.

$$\overline{\boldsymbol{A}}\, \boldsymbol{G} = \overline{\boldsymbol{W}} \tag{12}$$

with :

$$\overline{\boldsymbol{A}} = \begin{pmatrix} & \boldsymbol{I}_3 & & & \boldsymbol{I}_3 & & & \boldsymbol{I}_3 & \\ & \boldsymbol{B}_1 & & & \boldsymbol{B}_2 & & & \boldsymbol{B}_3 & \\ 1 & 0 & -k_{xz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -k_{xz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -k_{xz} \end{pmatrix}$$

$$\boldsymbol{G} = \begin{pmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \\ \boldsymbol{f}_3 \end{pmatrix} \qquad \overline{\boldsymbol{W}} = \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{M} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using some rows combination of the matrix $\overline{\boldsymbol{A}}$, Eq (12) can be written as :

$$\widehat{\boldsymbol{A}}\, \boldsymbol{G} = \widehat{\boldsymbol{W}} \tag{13}$$

With: $\widehat{\boldsymbol{A}} =$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -k_{xz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -k_{xz} \\ -P_{y,1} & P_{x,1} & 0 & -P_{y,2} & P_{x,2} & 0 \\ P_{z,1} & 0 & -P_{x,1} & P_{z,2} & 0 & -P_{x,2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -P_{z,1} & P_{y,1} & 0 & -P_{z,2} & P_{y,2} \end{pmatrix}$$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -k_{xz} \\ -P_{y,3} & P_{x,3} & 0 \\ P_{z,3} & 0 & -P_{x,3} \\ 0 & -P_{z,3} & P_{y,3} \end{matrix} \Bigg) \quad \widehat{\boldsymbol{W}} = \begin{pmatrix} F_X \\ F_Y \\ 0 \\ 0 \\ M_Z \\ M_Y \\ 0 \\ M_X \end{pmatrix}$$

where $\widehat{\boldsymbol{A}} \in \Re^{8\times 9}$ is the resulting matrix of $\overline{\boldsymbol{A}}$ after combination. $\boldsymbol{G} \in \Re^{8}$ is the robots force vector. $\widehat{\boldsymbol{W}} \in \Re^{8}$ is the resulting vector of $\boldsymbol{W}$ after combination. Thus, the force distribution problem is subjected to the inequality constraints expressed by (7), (8) (11).

4 Quadratic Problem Formulation and Solution

The solution to the inverse dynamic equations of this system is not unique, but it can be chosen in an optimal manner by minimizing some objective function. The approach taken here is to minimize the sum of the weighted torque of the robot, which results in the following objective function [15] and [21]:

$$f_G = \boldsymbol{p}^T \boldsymbol{G} + \frac{\boldsymbol{G}^T \boldsymbol{Q} \boldsymbol{G}}{2} \tag{14}$$

with:

$$\boldsymbol{p}^T = [\widehat{\tau}_1^T \boldsymbol{J}_1^T \,......., \widehat{\tau}_n^T \boldsymbol{J}_n^T] \quad \in \Re^{3n}$$

$$\boldsymbol{Q} = \begin{pmatrix} \boldsymbol{J}_1 \boldsymbol{q}_1 \boldsymbol{J}_1^T & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \boldsymbol{J}_n \boldsymbol{q}_n \boldsymbol{J}_n^T \end{pmatrix} \quad \in \Re^{3n\times 3n}$$

where $\widehat{\tau_j}$ is the joint torque vector due to the weight and inertia of the robot "j", $\mathbf{J}_j$ is the Jacobian of the robot "j", and $\mathbf{q}_j$ is a positive definite diagonal weighting matrix of the robot j. This objective function is strictly convex. Because the time for obtaining a solution does not depend on an initial guess, a quadratic programming is superior to linear programming in both speed and quality of the obtained solution [15]. The general linear-quadratic programming problem of the force distribution on robot is stated by :

$$\boldsymbol{p}^T \boldsymbol{G} + \frac{\boldsymbol{G}^T \boldsymbol{Q} \boldsymbol{G}}{2} \tag{15}$$

$$\widehat{\boldsymbol{A}} \widehat{\boldsymbol{G}} = \widehat{\boldsymbol{W}} \tag{16}$$

$$\boldsymbol{B} \widehat{\boldsymbol{G}} \leq \boldsymbol{C} \tag{17}$$

where $\mathtt{G} \in \Re^{9}$ is a vector of the design variables. It should be pointed out that, Eq. (19) denotes Eq (13), and Eq(20) is the resulting inequality constraints for the combination of Eq (7), Eq (8) and (11) where

3. $\boldsymbol{B} = [\boldsymbol{B}_1^T \, \boldsymbol{B}_2^T \, \boldsymbol{B}_3^T \, \boldsymbol{B}_4^T]^T \quad \in \Re^{9\times 24}$
$\boldsymbol{C} = [\tau_{1max} \,..\, \tau_{6max} \; -\tau_{1max} \,..\, -\tau_{6max} \; \boldsymbol{0\,0\,0\,0\,0\,0}]^T \quad \in \Re^{24}$

with

$$B_1 = \begin{bmatrix} \boldsymbol{J}_1^T \boldsymbol{R}_1 & 0 & 0 \\ 0 & \boldsymbol{J}_2^T \boldsymbol{R}_2 & 0 \\ 0 & 0 & \boldsymbol{J}_3^T \boldsymbol{R}_3 \end{bmatrix} \quad \in \Re^{9\times 9}$$

$$\boldsymbol{B}_2 = \begin{bmatrix} -\boldsymbol{J}_1^T \boldsymbol{R}_1 & 0 & 0 \\ 0 & -\boldsymbol{J}_2^T \boldsymbol{R}_2 & 0 \\ 0 & 0 & -\boldsymbol{J}_3^T \boldsymbol{R}_3 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 0 & 1 & -\mu^\star & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\mu^\star & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\mu^\star \end{bmatrix}$$

In Eq (19), we have eight linear independent equations with nine unknown variables. By using Gauss algorithm, this equation is transformed as follow :

$$[\boldsymbol{I}_8 \quad \widehat{\boldsymbol{A}}_r] \begin{bmatrix} \widehat{\boldsymbol{G}}_b \\ \widehat{\boldsymbol{G}}_r \end{bmatrix} = \widehat{\boldsymbol{W}}_r, \tag{18}$$

where $\boldsymbol{I}_8 \in \Re^{8\times 8}$ identity matrix, $\widehat{\boldsymbol{A}}_r \in \Re^{8}$ is the remaining column of the matrix $\widehat{\boldsymbol{A}}$ after transformation. $\widehat{\mathbf{G}}_b \in \Re^{8}$ is the partial vector of $\boldsymbol{G}$. $\widehat{\boldsymbol{G}}_r \in \Re$ is the unknown element of $\widehat{\boldsymbol{G}}$ which denotes the design variable. $\widehat{\boldsymbol{W}}_r \in \Re^{8}$ is the resulting vector of $\widehat{\boldsymbol{W}}$ after transformation. Equation(21) may be rewritten by the following form

$$\boldsymbol{I}_8 \widehat{\boldsymbol{G}}_b + \widehat{\boldsymbol{A}}_r \widehat{\boldsymbol{G}}_r - \widehat{\boldsymbol{W}}_r = 0, \tag{19}$$

Which yields to

$$\widehat{\boldsymbol{G}}_b = \widehat{\boldsymbol{W}}_r - \widehat{\boldsymbol{A}}_r \widehat{\boldsymbol{G}}_r. \tag{20}$$

Finally, it results in

$$\boldsymbol{G} = \begin{bmatrix} \widehat{\boldsymbol{G}}_b \\ \widehat{\boldsymbol{G}}_r \end{bmatrix} = \begin{bmatrix} \widehat{\boldsymbol{W}}_r \\ 0 \end{bmatrix} + \begin{bmatrix} -\widehat{\boldsymbol{A}}_r \\ 1 \end{bmatrix} \widehat{\boldsymbol{G}}_r. \tag{21}$$

Now let $\widehat{\boldsymbol{G}}_0 = [\widehat{\boldsymbol{W}}_r^T \quad 0]^T \in \Re^{8}$ *and* $N = [-\widehat{\boldsymbol{A}}_r^T \quad 1]^T \in \Re^{9}$, then Eq (24) becomes

$$\boldsymbol{G} = \widehat{\boldsymbol{G}}_0 + \boldsymbol{N} \widehat{\boldsymbol{G}}_r. \tag{22}$$

Substituting Eq (22) into Eqs (15) and (17), the linear quadratic programming problem can be expressed by :

$$minimize \quad f(\widehat{\boldsymbol{G}}_r), \tag{23}$$

$$subject\,to \quad \boldsymbol{B}\boldsymbol{N}\widehat{\boldsymbol{G}}_r \leq \boldsymbol{C} - \boldsymbol{B}\widehat{\boldsymbol{G}}_0. \tag{24}$$

where

$$f(\widehat{\boldsymbol{G}}_r) = \boldsymbol{p}^T \widehat{\boldsymbol{G}}_0 + \tfrac{1}{2}\widehat{\boldsymbol{G}}_0^T Q \widehat{\boldsymbol{G}}_0 + \boldsymbol{p}^T \boldsymbol{N}\widehat{\boldsymbol{G}}_r$$
$$+\tfrac{1}{2}\widehat{\boldsymbol{G}}_0^T \boldsymbol{QN}\widehat{\boldsymbol{G}}_r + \tfrac{1}{2}\widehat{\boldsymbol{G}}_r^T \boldsymbol{N}^T \boldsymbol{Q}\widehat{\boldsymbol{G}}_0$$
$$+\tfrac{1}{2}\widehat{\boldsymbol{G}}_r^T \boldsymbol{N}^T \boldsymbol{QN}\widehat{\boldsymbol{G}}_r$$

Since $\widehat{\boldsymbol{G}}_r$ is a single variable
denoted by x, the optimal force distribution can be written as :

$$minimize \quad a_0 x^2 + a_1 x + a_2 \qquad subject\, to \quad x \in [b_1\, b_2] \tag{25}$$

With

$$a_0 = \tfrac{1}{2}\boldsymbol{N}^T \boldsymbol{QN}$$
$$a_1 = \boldsymbol{p}^T \boldsymbol{N} + \tfrac{1}{2}\widehat{\boldsymbol{G}}_0^T \boldsymbol{QN} + \tfrac{1}{2}\boldsymbol{N}^T \boldsymbol{Q}\widehat{\boldsymbol{G}}_0$$
$$a_2 = \boldsymbol{p}^T \widehat{\boldsymbol{G}}_0 + \tfrac{1}{2}\widehat{\boldsymbol{G}}_0^T \boldsymbol{Q}\widehat{\boldsymbol{G}}_0$$

Where $[b_1\, b_2]$ denotes the bound resulted from Eq(24). Since it is clear that $a_0 \geq 0$ because of the positive-definite matrix Q, There must be an optimal solution for the force distribution problem.

5 Simulations

The basic mechanism, size and parameters of the object and one robot are shown in Figure (1) and (2), where a = 0.25 [m], b = 0.61 [m], $l_1 = 0.05$ [m], $l_2 = 0.20$ [m], $l_3 = 0.30$ [m] and $l_4 \simeq 0$ [m]. There are six actuated joints $\theta_{1,j}$, $\theta_{2,j}$, $....and\, \theta_{6,j}$ in the robot "j", whose torques are denoted as $\tau_{1,j}\ \tau_{2,j}\ ...and\, \tau_{6,j}$, for (j=1,2,3), respectively. The Jacobian of the robot "j" can be expressed by.

$$\boldsymbol{J}_j^T = [\boldsymbol{J}_{j,1} \quad \boldsymbol{J}_{j,2} \quad \boldsymbol{J}_{j,3}] \tag{26}$$

for (j=1,2,3), where

$$\boldsymbol{J}_{j,1} = \begin{bmatrix} -S(\theta_{1,j})C(\theta_{2,j})(R4 + l_2 + r3) \\ -C(\theta_{1,j})S(\theta_{2,j})(R4 + l_2 + r3) \\ C(\theta_{1,j})C(\theta_{2,j}) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{J}_{j,2} = \begin{bmatrix} C(\theta_{1,j})C(\theta_{2,j})(R4 + l_2 + r3) \\ -S(\theta_{1,j})S(\theta_{2,j})(R4 + l_2 + r3) \\ S(\theta_{1,j})C(\theta_{2,j}) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{J}_{j,3} = \begin{bmatrix} 0 \\ -C(\theta_{2,j})(R4 + l_2 + r3) \\ S(\theta_{2,j}) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From Figure (4) the orientation matrix of $(x_{6,j}, y_{6,j}, z_{6,j})$ with respect to the frame (x_0, y_0, z_0) can be obtained by,

$$\boldsymbol{A}_{0j} = \begin{pmatrix} Cos\,\phi_{1,j} & Sin\,\phi_j & 0 \\ -Sin\,\phi_j & Cos\,\phi_j & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{27}$$

5.1 Simulation results

In order to show the effectiveness of proposed approach,we consider three identical 6-DOF manipulators grasping a rigid object ,the base of each robot is located at a distance of a_1 see figure(6),We denote $X_i \in \Re^6$ where : $X_i = [P_{xi}, P_{yi}, P_{zi}, \theta_i, \phi_i, \psi_i]^T$ represents the vector Cartesian position and orientation of the end-effector of robot i . Furthermore the load vector coordinates (force tensor) at the object is $F = [-5, 10, -250, 3, 1, 2]^T$, $\mu = 0.05$ the static coefficient of friction
some simulations were conducted under Matlab. We consider that the object rotate on the axis and simultaneous deplaced on Z_0.

For the objective function Eq (14), the weighting matrix are choosen as follow : $\boldsymbol{p} = \boldsymbol{0}$ and $\boldsymbol{Q} = \boldsymbol{I}$ (the identity matrix).
The figure 6 shows the trajectory of the object in the X-Y-Z suported with the three robots. The associated joints coordinates are obtained by using the direct and inverse geometric model (Eq (2)and Eq (3)). In the figures 7-9, the force distribution of the object are given. We can show that, this distribution validate the following force equilbrium equation :

$$\Sigma f x_j = Fx, \quad ,\Sigma f y_j = Fy, \quad \Sigma f z_j = Fz$$

Elsewhere, the z-force components fz_j are never negative, respecting the contact constraint. We can also show that the constraint Eq (6) are always satisfiyed as shown in figure 10 for the robot(j) ,j=1,2,3 where the curve $fx_j^2 + fy_j^2$ is always under the curve $\mu^2 fz_j^2$

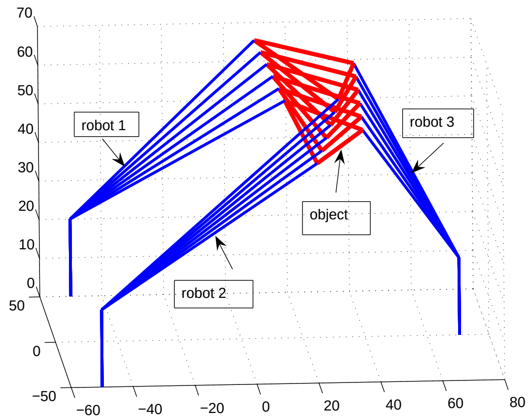

Figure 6: **View of the object suported by three robots**

Figure 7: **Forces on the first robot**

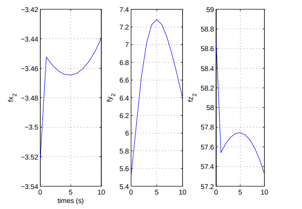

Figure 8: **Forces on the second robot**

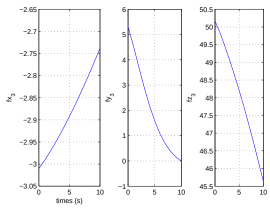

Figure 9: **Forces on the third robot**

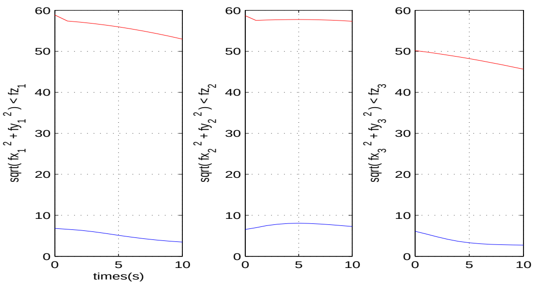

Figure 10: Constraint $fx_j^2 + fy_j^2 \leq \mu^2 fz_j^2$ satisfaction

6 Conclusion

In this paper, the authors have presented the force distribution problem in the case of multiple manipulators system. First, the robot inverse and direct geometric models where presented. Then, the real time force distribution problem where formulated in terms of non linear programming problem. After problem size reduction and transformation, the initial problem is solved in terms of quadratic programming problem. Simulations results where presented. Actually, we are working on the generalisation of this approach at $n > 3$ robots case. Finaly, a new learning approach is under developpment for real-time operationnal space control of the system.

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