MODELING AND SIMULATION OF SEVERE SLUGGING IN AIR-WATER SYSTEMS INCLUDING INERTIAL EFFECTS

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ABSTRACT
A mathematical model and numerical simulations corresponding to severe slugging in air-water pipeline-riser systems, are presented. The mathematical model considers continuity equations for liquid and gas phases, with a simplified momentum equation for the mixture. A drift-flux model, evaluated for the local conditions in the riser, is used as a closure law. In many models appearing in the literature, propagation of pressure waves is neglected both in the pipeline and in the riser. Besides, variations of void fraction in the stratified flow in the pipeline are also neglected and the void fraction obtained from the stationary state is used in the simulations. This paper shows an improvement in a model previously published by the author, including inertial effects. In the riser, inertial terms are taken into account by using the rigid water-hammer approximation. In the pipeline, the local acceleration of the water and gas phases are included in the momentum equations for stratified flow, allowing to calculate the instantaneous values of pressure drop and void fraction. The developed model predicts the location of the liquid accumulation front in the pipeline and the liquid level in the riser, so it is possible to determine which type of severe slugging occurs in the system. A comparison is made with experimental results published in literature including a choke valve and gas injection at the bottom of the riser, showing very good results for slugging cycle and stability maps.

Keywords: severe slugging, pipeline-riser system, air-water flow, stability, lumped and distributed parameter systems, switched systems, petroleum production technology

1. INTRODUCTION

Severe slugging is a terrain dominated phenomenon, characterized by the formation and cyclical production of long liquid slugs and fast gas blowdown. Severe slugging may appear for low gas and liquid flow rates when a section with downward inclination angle (pipeline) is followed by another section with an upward inclination (riser). This configuration is common in off-shore petroleum production systems. Main issues related to severe slugging are: a) High average back pressure at well head, causing tremendous production losses, b) High instantaneous flow rates, causing instabilities in the liquid control system of the separators and eventually shutdown, and c) Reservoir flow oscillations.

For steady state and low flow rates, the flow pattern in the pipeline may be stratified, while it may be intermittent in the riser, as shown in Fig. 1(a).

A cycle of severe slugging can be described as taking place according to the following stages (Taitel, 1986). Once the system destabilizes and gas passage is blocked at the bottom of the riser, liquid continues to flow in and gas already in the riser continues to flow out, being possible that the liquid level in the riser falls below the top level at the separator. As a consequence, the riser column becomes heavier and pressure at the bottom of the riser increases, compressing the gas in the pipeline and creating a liquid accumulation region. This stage is known as slug formation (Fig. 1(b)).

As the liquid level reaches the top while the gas passage is kept blocked at the bottom, pressure reaches a maximum and there is only liquid flowing in the riser. This is the slug production stage (Fig. 1(c)).

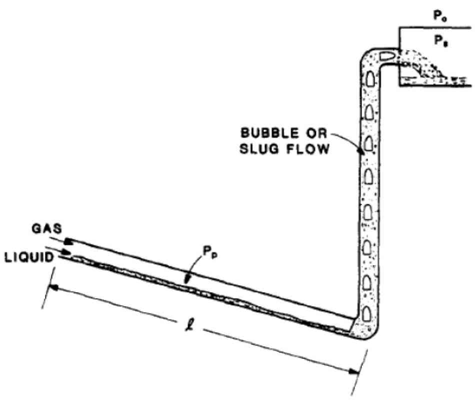
Since gas keeps flowing in the pipeline, the liquid accumulation front is pushed back until it reaches the bottom of the riser, starting the blowout stage (Fig. 1(d)).

As the gas phase penetrates into the riser the column becomes lighter, decreasing the pressure and then rising the gas flow. When gas reaches the top of the riser, gas passage is free through the stratified flow pattern in the pipeline and the intermitent/annular flow pattern in the riser, causing a violent expulsion and a rapid decompression that brings the process to slug formation again. This stage is known as gas blowdown (Fig. 1(e)).

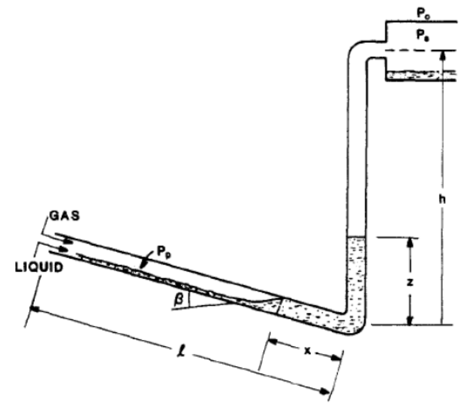
Figure 1(f) shows the different stages in the pressure history at the bottom of the riser corresponding to an experiment under laboratory conditions (Schmidt, 1977).

A classification of severe slugging can be made, according to the observed flow regime, as follows:

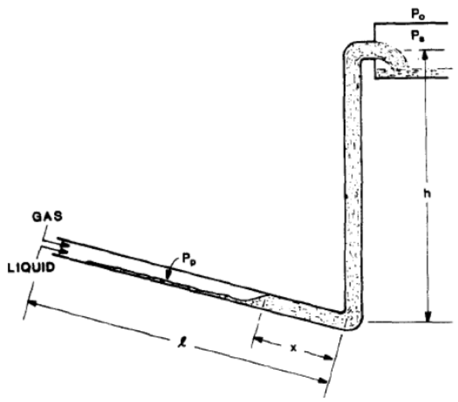
- Severe Slugging 1 (SS1): the liquid slug length is greater to or equal to one riser length and maximum pipeline pressure is equal to the hydrostatic head of the riser (neglecting friction pressure drop).
- Severe Slugging 2 (SS2): the liquid length is less than one riser length, with intermittent gas penetration at the bottom of the riser.
- Severe Slugging 3 (SS3): there is continuous gas penetration at the bottom of the riser; visually, the flow in the riser resembles normal slug flow, but pressure, slug lengths and frequencies reveal cyclic variations of smaller periods and amplitudes compared to SS1.



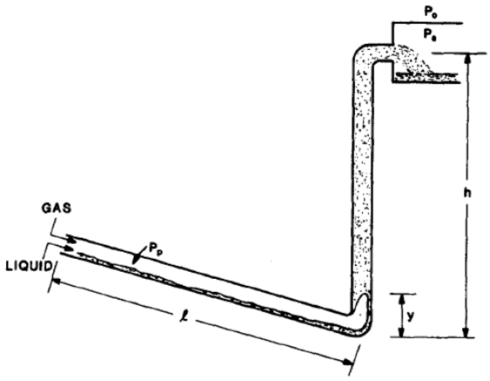
(a) Steady state.



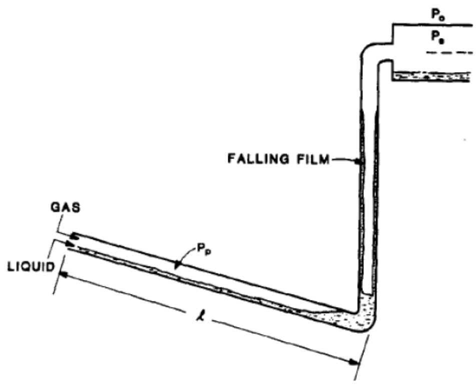
(b) Slug formation.



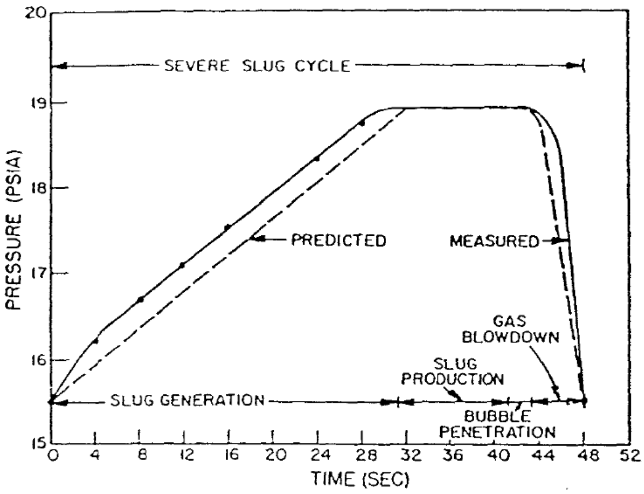
(c) Slug production.



(d) Gas blowout.



(e) Gas blowdown.



(f) Pressure history.

Figure 1: Stages for severe slugging (from (Taitel, 1986) and (Schmidt, 1977)).

- Oscillation (OSC): there are cyclic pressure fluctuations without the spontaneous vigorous blowdown.

Most of the models for severe slugging were developed for vertical risers and assume one-dimensional, isothermal flow and a mixture momentum equation in which only the gravitational term is important.

In (Taitel *et al.*, 1990) a model was presented considering constant mean values for the gas density and void fraction in the riser, allowing to calculate time variations of pipeline pressure, position of the accumulation region, flow rate into the riser and mean holdup. It was found that as the operation point moves closer to the stability line the numerical procedure did not converge, giving gas mass flows going to infinite as the spatial discretization was decreased. Experimental data were obtained from a facility for different buffer volumes (simulating equivalent pipeline lengths) and a comparison was made with the simulation results, showing good agreement except for the blowout/blowdown stage. Setting apart the non-convergence problems, lumped parameter models seem to work fine for short risers, where the local variations of variables are small, but are not successful in long risers, typical of offshore systems.

In (Sarica and Shoham, 1991) a model with a distributed parameter formulation for the riser was presented. Considering continuity equations for the liquid and gas without phase change and a gravity-dominant mixture momentum equation, the model was capable of handling discontinuities such as liquid accumulation in the piping and liquid level in the riser. The resulting equations were solved by using the method of characteristics. A comparison of simulations with different experimental data showed reasonable agreement, although the model also suffered from non-convergence in the unstable region.

In (Baliño *et al.*, 2010) a model for severe slugging valid for risers with variable inclination was presented. The model was used to simulate numerically the air-water multiphase flow in a catenary riser for the experimental conditions reported in (Wordsworth *et al.*, 1998). Stability and flow regime maps in the system parameter space for the multiphase flow in a catenary pipeline-riser system were built.

In (Nemoto and Baliño, 2012) the model developed in (Baliño *et al.*, 2010) was extended to investigate the dynamics of gas, oil and water flow in a pipeline-riser system. Mass transfer between the oil and gas phases was calculated using the black oil approximation. The properties of fluids were calculated by analytical correlations based on experimental results and field data.

The stationary solution for a given point in the system parameter space is given as initial condition for the numerical simulation; if the numerical solution does not go away from the initial condition with time, the stationary solution is stable and it is the system steady state. If the numerical solution goes away with time, the stationary state is unstable, there is no steady state and an intermittent solution develops with time. By changing the point in the system parameter space and repeating this process, the stability map can be built. For unstable flow, the analysis of the limit cycle leads to the determination of the flow regime map, showing the regions corresponding to the different types of intermittency.

In all the models reviewed above, propagation of pressure waves is neglected both in the pipeline and in the riser; this constitutes the no-pressure-wave (NPW) approximation (Masella *et al.*, 1998). As a result of the NPW approximation, pressure changes are felt instantaneously at any point in the domain (pressure waves travel at infinite speed).

Also in the models reviewed above, a stratified flow pattern is assumed at the pipeline and variations of void fraction are neglected. The void fraction is obtained from a momentum balance in the gas and liquid phases, resulting an algebraic relation between the mean variables (Taitel and Dukler, 1976). The void fraction determined in the stationary state is assumed as constant in the simulations, so the momentum balance equation is not satisfied and variations of the void fraction cannot be calculated in the transients.

This paper shows an improvement in the model previously published by the author (Baliño *et al.*, 2010), including inertial effects.

In the riser, inertial terms are taken into account by using the rigid water-hammer approximation (Chaudhry, 1987). In this approximation, the acceleration terms for the liquid and gas phases are taken into account in the momentum equation, but compressibility of the liquid phase is neglected in the mass conservation equation. In the pipeline, convective acceleration terms are neglected but the local acceleration terms for the water and gas phases are included in the momentum equations for stratified flow, allowing to calculate the instantaneous values of pressure drop and void fraction.

The model includes additional devices, such as a valve located at the top of the riser and a gas injection line at the bottom of the riser. In this way it is possible to evaluate valve closure and gas lift as mitigating actions for severe slugging.

2. MODEL

The model considers one-dimensional flow in both pipeline and riser subsystems. The liquid phase is assumed incompressible, while the gas phase is considered as an ideal gas. Both phases flow in isothermal conditions. The flow pattern in the pipeline is assumed as stratified. The model is capable of handling discontinuities in the flow, such as liquid accumulation in the pipeline, liquid level in the riser and void fraction waves.

2.1 Pipeline

The pipeline, shown in Fig. 2, can be either in a condition of liquid accumulation ($x > 0$) or in a condition of continuous gas penetration ($x = 0$), where x is the position of the liquid accumulation front. The existence of a buffer vessel with volume υ_e is considered in order to simulate an equivalent pipeline length $L_e = \frac{\upsilon_e}{A}$, where A is the flow passage area ($A = \frac{1}{4}\,\pi\,D^2$, where D is the inner diameter).

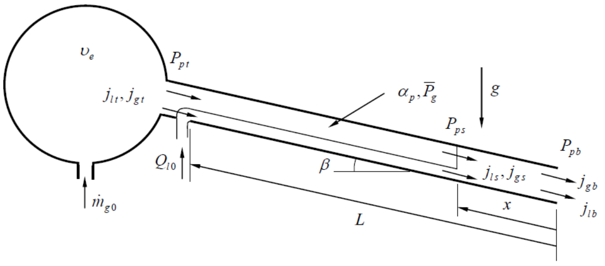


Figure 2: Definition of variables at the pipeline.

Three locations in the pipeline can be identified, with corresponding pressures and superficial velocities for the liquid and gas phases: the bottom ($P_{p\,b}$, $j_{l\,b}$ and $j_{g\,b}$), the stratified region located at the liquid accumulation front ($P_{p\,s}$, $j_{l\,s}$ and $j_{g\,s}$) and the top ($P_{p\,t}$, $j_{l\,t}$ and $j_{g\,t}$).

Mass and momentum conservation equations are applied for the control volumes corresponding to the liquid and gas phases. In the mass equation, a mean pressure $\overline{P}_g(t) = \frac{1}{2}\,(P_{p\,t} + P_{p\,s})$ is assumed. In mass and momentum equations, variations of the void fraction with position are neglected and a mean void fraction $\alpha_p(t)$ is considered. These approximations are made in order to get a lumped parameter model for the pipeline.

The superficial velocities at the top of the pipeline can be written as:

$$j_{l\,t} = \frac{Q_{l\,0}}{A} \tag{1}$$

$$j_{g\,t} = \frac{\dot{m}_{g\,0}\,R_g\,T_g}{P_{p\,t}\,A} \tag{2}$$

where $\dot{m}_{g\,0}$ and $Q_{l\,0}$ are respectively the gas mass flow and the liquid volumetric flow injected in the pipeline and R_g and T_g are respectively the gas constant and temperature.

2.1.1 Condition $x > 0$

For this condition there is no gas flowing out the pipeline, resulting:

$$j_{g\,b} = 0 \tag{3}$$

Applying mass conservation equation for the liquid and gas phases, it is obtained:

$$\frac{d\alpha_p}{dt} = \frac{j_{l\,b} - \frac{Q_{l\,0}}{A} + \alpha_p\,\frac{dx}{dt}}{L - x} \tag{4}$$

$$\frac{d\overline{P}_g}{dt} = \frac{-\overline{P}_g\left(j_{l\,b} - \frac{Q_{l\,0}}{A}\right) + \frac{R_g\,T_g}{A}\,\dot{m}_{g\,0}}{(L - x)\,\alpha_p + L_e} \tag{5}$$

where L is the pipeline length and t is time. A mass balance for the liquid in the stratified region yields:

$$-\,(L - x)\,\frac{d\alpha_p}{dt} + j_{l\,s} - \frac{Q_{l\,0}}{A} = 0 \tag{6}$$

From Eq. (4) and (6), it results:

$$j_{l\,s} = j_{l\,b} + \alpha_p\,\frac{dx}{dt} \tag{7}$$

From the kinematic condition at the liquid penetration front, we get:

$$j_{g\,s} = -\alpha_p\,\frac{dx}{dt} \tag{8}$$

Applying the momentum conservation equation in the control volume of liquid region at the penetration front and including friction and inertial terms, we get:

$$P_{p\,b} = P_{p\,s} + \rho_l\,x\left(g\,\sin\beta - \frac{2\,f_{ll}}{D}\,j_{l\,b}\,|j_{l\,b}| - \frac{dj_{l\,b}}{dt}\right) \tag{9}$$

where f_{ll} is the Fanning friction factor (assuming that only liquid is filling the cross sectional area), g is the gravity acceleration constant, ρ_l is the liquid density and β is the pipeline inclination angle (positive when downwards). The Fanning friction factor is calculated by using the correlation from (Chen, 1979).

2.1.2 Condition $x = 0$

For this condition there is no liquid penetration front, resulting:

$$j_{g\,s} = j_{g\,b} \tag{10}$$

$$j_{l\,s} = j_{l\,b} \tag{11}$$

$$P_{p\,s} = P_{p\,b} \tag{12}$$

Applying mass conservation equation for the liquid and gas phases, it is obtained:

$$\frac{d\alpha_p}{dt} = \frac{1}{L}\left(j_{l\,b} - \frac{Q_{l\,0}}{A}\right) \tag{13}$$

$$\frac{d\overline{P}_g}{dt} = \frac{-\overline{P}_g\left(j_{g\,b} + j_{l\,b} - \frac{Q_{l\,0}}{A}\right) + \frac{R_g\,T_g}{A}\,\dot{m}_{g\,0}}{L\,\alpha_p + L_e} \tag{14}$$

2.1.3 Local equilibrium condition for stratified flow

In previous models, the void fraction at the pipeline is determined from an algebraic relationship evaluated from the stationary state, derived from the momentum balance in stratified flow (Taitel and Dukler, 1976). This relationship can be generalized by including the local inertial terms. Assuming stratified flow (see Fig. 3), the generalized relationship can be written as:

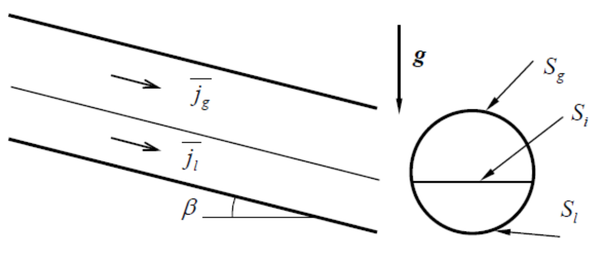


Figure 3: Stratified flow at the pipeline.

$$
\tau_{w\,g}\,\frac{S_g}{\alpha_p} - \tau_{w\,l}\,\frac{S_l}{1-\alpha_p} + \tau_i\,S_i\left(\frac{1}{1-\alpha_p}+\frac{1}{\alpha_p}\right)
$$
$$
+\left(\rho_l - \overline{\rho}_g\right)\,A\,g\,\sin\beta - A\left(\rho_l\,\frac{d\overline{u}_l}{dt} - \overline{\rho}_g\,\frac{d\overline{u}_g}{dt}\right) = 0 \tag{15}
$$

$$
\overline{u}_g = \frac{\overline{j}_g}{\alpha_p} \tag{16}
$$

$$
\overline{u}_l = \frac{\overline{j}_l}{1-\alpha_p} \tag{17}
$$

$$
\overline{\rho}_g = \frac{\overline{P_g}}{R_g\,T_g} \tag{18}
$$

$$
\overline{j}_g = \frac{1}{2}\,\left(j_{g\,t} + j_{g\,s}\right) \tag{19}
$$

$$
\overline{j}_l = \frac{1}{2}\,\left(\frac{Q_{l\,0}}{A} + j_{l\,s}\right) \tag{20}
$$

where $\overline{j}_g$ and $\overline{j}_g$ are respectively the mean superficial velocities for gas and liquid, S_g, S_i and S_l are respectively the gas, interfacial and liquid wetted perimeters, $\tau_{w\,g}$, τ_i and $\tau_{w\,l}$ are respectively the wall-gas, interface and wall-liquid shear stresses, $\overline{u}_g$ and $\overline{u}_g$ are respectively the mean phase velocities for gas and liquid and ρ_g is the gas density.

In Eq. (15) the wetted and interfacial perimeters are determined considering a stratified geometry, while the shear stresses are related to the superficial velocities of the phases through friction factors based on the hydraulic diameters for each phase (Kokal and Stanislav, 1989).

2.2 Riser

Continuity equations for the phases are considered at the riser (see Fig. 4). This results in the following set of equations:

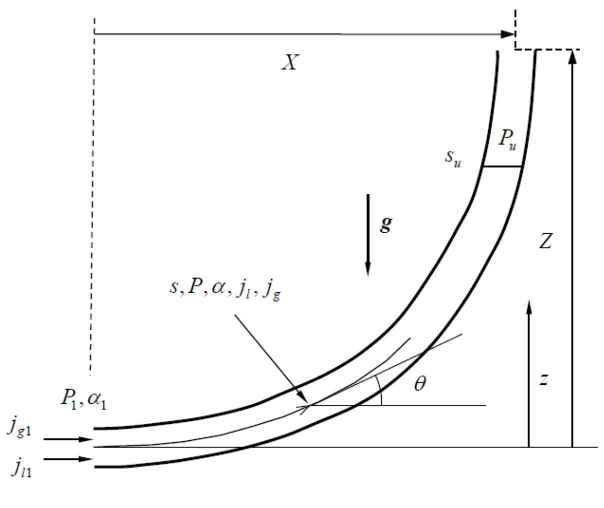


Figure 4: Definition of variables at the riser.

$$
-\frac{\partial\alpha}{\partial t} + \frac{\partial j_l}{\partial s} = 0 \tag{21}
$$

$$
\frac{\partial}{\partial t}\,(P\,\alpha) + \frac{\partial}{\partial s}\,(P\,j_g) = 0 \tag{22}
$$

where j_g, j_l and j are respectively the gas, liquid and total superficial velocities, P is the pressure, s is the position along the riser, α is the void fraction and u_g and u_l are respectively the gas and liquid phase velocities.

The superficial velocities for the phases are determined by using a drift flux correlation, assumed to be locally valid:

$$
j_g = u_g\,\alpha = \alpha\,(C_d\,j + U_d) \tag{23}
$$

$$
j_l = j - j_g = u_l\,(1-\alpha) = (1-\alpha\,C_d)\,j - \alpha\,U_d \tag{24}
$$

It will be assumed that the drift parameters C_d and U_d depend at most on the local flow conditions and inclination angle $\theta = \theta\,(s)$, this is, $C_d = C_d\,(\alpha,\,P,\,j,\,\theta)$ and $U_d = U_d\,(\alpha,\,P,\,j,\,\theta)$ (Bendiksen, 1984; Chexal *et al.*, 1992).

The drift coefficients used in the model are (Bendiksen, 1984):

- For $Fr_j < 3.5$:

$$
C_d = 1.05 + 0.15\,\sin\,\theta \tag{25}
$$

$$
U_d = \sqrt{g\,D}\,(0.35\,\sin\,\theta + 0.54\,\cos\,\theta) \tag{26}
$$

- For $Fr_j \geq 3.5$:

$$
C_d = 1.2 \tag{27}
$$

$$
U_d = 0.35\,\sqrt{g\,D}\,\sin\,\theta \tag{28}
$$

where the Froude number Fr_j is defined as:

$$
Fr_j = \frac{j}{\sqrt{g\,D}} \tag{29}
$$

Considering as the state variables in the riser the void fraction, pressure and total superficial velocity (functions of position and time), Eq. (21) and (22) can be finally rewritten as:

$$
\frac{D_g\alpha}{Dt} + \alpha\,\frac{\partial}{\partial s}\,(C_d\,j + U_d) - \frac{\partial j}{\partial s} = 0 \tag{30}
$$

$$
\alpha\,\frac{D_gP}{Dt} + P\,\frac{\partial j}{\partial s} = 0 \tag{31}
$$

where:

$$
\frac{D_g}{Dt} = \frac{\partial}{\partial t} + u_g\,\frac{\partial}{\partial s} \tag{32}
$$

A mixture momentum equation is considered, in which the inertial terms corresponding to the liquid and gas phases were included :

$$
\frac{\partial P}{\partial s} = -\rho_m\left(g\,\sin\theta + 2\,\frac{f_m}{D}\,j\,|j|\right)
$$
$$
-\alpha\,\rho_g\,\frac{D_g u_g}{Dt} - (1-\alpha)\,\rho_l\,\frac{D_l u_l}{Dt} \tag{33}
$$

$$
\rho_m = \rho_l\,(1-\alpha) + \frac{P}{R_g\,T_g}\,\alpha \tag{34}
$$

$$
\frac{D_l}{Dt} = \frac{\partial}{\partial t} + u_l\,\frac{\partial}{\partial s} \tag{35}
$$

where f_m is the Fanning friction factor (dependent on the Reynolds number and the relative roughness ϵ/D, where ϵ is the pipe roughness), calculated from (Chen, 1979) using a homogeneous mixture two-phase model and ρ_m is the mixture density.

2.2.1 Gas lift

Gas lift is a process used to artificially lift fluids from wells where there is insufficient reservoir pressure. By injecting gas, it is possible to aerate the liquid column and to reduce the pressure gradient. Due to this effect, it is acknowledged that gas lift can stabilize the flow in severe slugging, although relatively large gas flow rates are necessary (Jansen *et al.*, 1996). In the model it is considered the possibility of injecting gas in a position s_l along the riser. Considering the balance conservation equations with a gas mass source term, it can be shown that the gas injection introduces the following discontinuities in position in the gas superficial velocity, pressure and void fractions:

$$j_g^+ = \frac{\frac{R_g\,T_g}{A}\,\dot{m}_{g\,l} + P^-\,j_g^-}{P^+} \tag{36}$$

$$P^- = \frac{P^+\left[1 - \frac{1}{R_g\,T_g}\left(\frac{j_g^2}{\alpha}\right)^+\right] - \rho_l\,j_l^2\left(\frac{1}{1-\alpha^+} - \frac{1}{1-\alpha^-}\right)}{1 - \frac{1}{R_g\,T_g}\left(\frac{j_g^2}{\alpha}\right)^-} \tag{37}$$

$$\alpha^+ = \frac{j_g^+}{\left(C_d\,j + U_d\right)^+} \tag{38}$$

where $\dot{m}_{g\,l}$ is the injected gas lift mass flow and the superscripts $-$ and $+$ denote evaluation respectively at location upstream and downstream of position s_l.

2.2.2 Gas region

In transients where the liquid level falls below the top of the riser ($s_u < s_t$) a gas region is formed, as shown in Fig. 5. This region is modeled considering a constant mean pressure $\overline{P}_{g\,r} = \frac{1}{2}\left(P_t + P_u\right)$ for the mass balance equation and friction, gravitational and inertial terms in the momentum balance equation; P_t and P_u are respectively the pressure at the top and at the liquid level in the riser. The resulting equations for the mass balance depends on the location of the gas lift injection s_l compared to the liquid level position s_u.

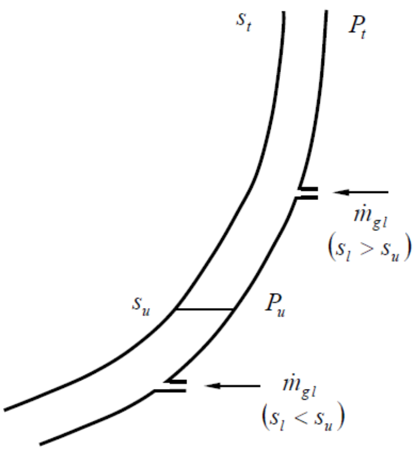


Figure 5: Definition of variables at the gas region.

If $s_u > s_l$, we get:

$$\frac{d\overline{P}_{g\,r}}{dt} = -\frac{\overline{P}_{g\,r}}{s_t - s_u}\left(j_{g\,t} - j_u\right) \tag{39}$$

If $s_u > s_l$, we get:

$$\frac{d\overline{P}_{g\,r}}{dt} = -\frac{\overline{P}_{g\,r}}{s_t - s_u}\left(j_{g\,t} - j_u - \frac{R_g\,T_g}{\overline{P}_{g\,r}\,A}\,\dot{m}_{g\,l}\right) \tag{40}$$

where $j_{g\,t}$ is the gas superficial velocity at the top of the riser, j_u is the total superficial velocity at the liquid level and s_t is the position of the top of the riser.

The momentum conservation equation results, for both cases:

$$P_u = P_t + \overline{\rho}_g\left[g\left(z_t - z_u\right) + \left(\frac{2\,f_{gg}}{D}\,\overline{j}_{g\,r}\,\left|\overline{j}_{g\,r}\right| - \frac{d\overline{j}_{g\,r}}{dt}\right)\left(s_t - s_u\right)\right] \tag{41}$$

$$\overline{j}_{g\,r} = \frac{1}{2}\left(j_u + j_{g\,t}\right) \tag{42}$$

where f_{gg} is the Fanning friction factor considering only gas flowing, $\overline{j}_{g\,r}$ and $\overline{\rho}_g$ are respectively the mean gas superficial velocity and the mean gas density at the gas region; z_t and z_u are respectively the vertical positions at the top of the riser and at the liquid level.

2.2.3 Riser geometry

The riser geometry is characterized by the coordinates X and Z corresponding to the top of the riser and a set of functions furnishing the ordinate z and local inclination angle θ as a function of the local position s along the riser (see Fig. 4). For a constant angle riser, for instance, it results:

$$\theta = \arctan\left(\frac{Z}{X}\right) \tag{43}$$

$$s_t = \left(X^2 + Z^2\right)^{1/2} \tag{44}$$

$$z = s\,\sin\theta \tag{45}$$

The definition of geometry for a catenary riser can be seen in (Baliño *et al.*, 2010).

2.2.4 Choke valve

In normal operation in petroleum production systems the choke valve controls the flow, allowing a production compatible with the reservoir characteristics. In severe slugging, it is acknowledged that choking can stabilize the flow by increasing the back pressure (Schmidt, 1977; Taitel, 1986). For low pressures, typical of air-water laboratory systems, the valve operates in subcritical condition; in this case, the flow depends on the pressure difference across the valve (see Fig. 6. The model considers a valve characteristic based on the homogeneous flow model:

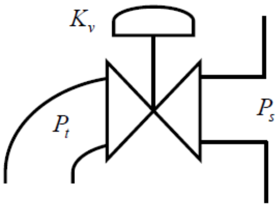


Figure 6: Definition of variables at the choke valve.

$$P_t - P_s = K_v \, \frac{1}{2} \, \rho_{m\,t} \, j_t \, |j_t| \tag{46}$$

$$\rho_{m\,t} = \alpha_t \, \rho_{g\,t} + (1 - \alpha_t) \, \rho_l \tag{47}$$

where j_t, α_t, $\rho_{m\,t}$, $\rho_{g\,t}$ are respectively the total superficial velocity, void fraction, mixture density and gas density, all of them evaluated at the top of the riser, P_s is the separator pressure and K_v is the valve constant.

Another expression for the pressure drop across the valve was used in (Jansen *et al.*, 1996):

$$P_t - P_s = C \, j_{l\,t} \, |j_{l\,t}| \tag{48}$$

where $j_{l\,t}$ is the liquid superficial velocity at the top of the riser and C is a dimensional valve constant. According to this relationship, it is neglected the contribution of the gas phase to pressure drop.

2.3 Coupling between pipeline and riser

Assuming the same flow passage area for the pipeline and riser, the pressure and superficial velocities at the bottom of the riser are continuous:

$$P\,(s = 0,\, t) = P_b\,(t) \tag{49}$$

$$j_g\,(s = 0,\, t) = j_{g\,b}\,(t) \tag{50}$$

$$j_l\,(s = 0,\, t) = j_{l\,b}\,(t) \tag{51}$$

The boundary condition for the void fraction can be obtained from Eq. (23) evaluated at the bottom of the riser:

$$\alpha\,(s = 0,\, t) = \alpha_b\,(t) = \frac{j_{g\,b}}{C_{d\,b}\, j_b + U_{d\,b}} \tag{52}$$

Figure 7 shows the state variables and the coupling between the subsystems. State variables for the pipeline are the average gas pressure, void fraction and position of the liquid accumulation front, while for the riser they are the local pressure, void fraction and total superficial velocity. The pipeline imposes the superficial velocities for the gas and liquid phases at the bottom of the riser, while the riser imposes the pressure to the pipeline; these variables are the boundary conditions for the corresponding subsystems. Additional boundary conditions are the liquid volumetric flow rate and the gas mass flow rate at the pipeline, as well as the gas lift mass flow rate and separation pressure at the riser.

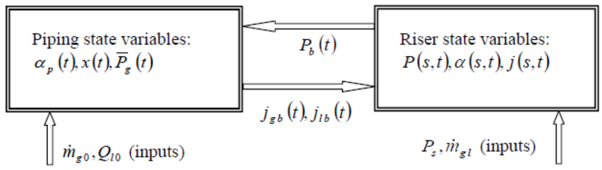


Figure 7: Coupling between subsystems.

As initial conditions, the stationary conditions were chosen, this is, the solution of the system of equations obtained after setting equal to zero the time derivatives.

3. DISCRETIZATION AND NUMERICAL IMPLEMENTATION

The system of equations corresponding to the stationary state, as well as the system of dynamic equations, were discretized and numerically implemented using the software MATLAB (Magrab *et al.*, 2005).

In the riser a moving grid method was adopted (see Fig. 8), in which node i $(1 \le i \le N-1)$ moves with the corresponding gas velocity (red lines), in order to calculate the directional derivatives of Eq. (32). Last node N moves with the liquid velocity if the liquid level falls below the top of the riser ($s_u < s_t$), or remains fixed at position s_t otherwise. The time step Δt^{k+1} is chosen as the time step such that the characteristic propagated from the $N-1$ th node intersects the position s_u at time $t^k + \Delta t^{k+1}$ if the liquid level falls below the top level in the riser, or as the time step such that the characteristic propagated from the $N-1$ th node intersects position s_t otherwise. Values at time t^k are interpolated in order to calculate the directional derivatives of Eq. (35), corresponding to the liquid velocity (blue lines). An implicit scheme was used, with a predictor-corrector method for treatment of the nonlinearities. Details of the procedure can be seen in (Baliño *et al.*, 2010).

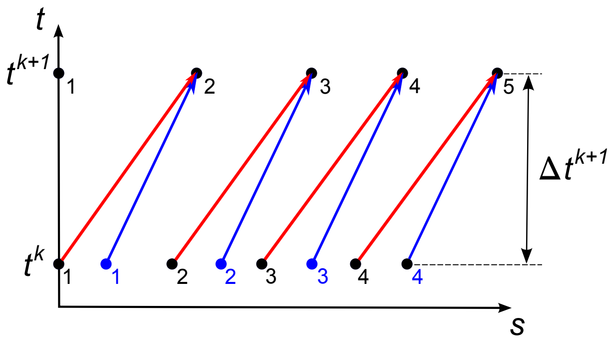


Figure 8: Discretization along the characteristic directions.

4. SIMULATIONS

In this Section, simulations corresponding to experimental data for a vertical riser are shown, considering the effects of the choke valve and gas lift injection at the bottom of the riser.

After a nodalization study, the riser was discretized in $N = 21$ nodes. Input flow variables are defined in terms of the superficial velocities $j_{g\,0}$ and $j_{l\,0}$ at standard conditions (pressure $P_0 = 1.013\,bara$, temperature $T_0 = 293\,K$); these superficial velocities are related to the flows as:

$$j_{g\,0} = \frac{R_g\,T_0\,\dot{m}_{g\,0}}{P_0\,A} \quad (53)$$

$$j_{l\,0} = \frac{Q_{l\,0}}{A} \quad (54)$$

4.1 Data from (Taitel *et al.*, 1990)

The following parameters were chosen for a comparison with experimental data of (Taitel *et al.*, 1990): fluid parameters are $\mu_l = 1. \times 10^{-3}\,kg/m/s$, $\mu_g = 1.8 \times 10^{-5}\,kg/m/s$, $\rho_l = 1. \times 10^{3}\,kg/m^3$, $R_g = 287\,m^2/s^2/K$ and $T_g = 293\,K$; pipeline parameters are $L = 9.1\,m$, $L_e = 1.69\,m$ and $\beta = 5\,^o$; riser height is $Z = 3\,m$; common parameters for pipeline and riser are $D = 2.54 \times 10^{-2}\,m$ and $\epsilon = 1.5 \times 10^{-6}\,m$; separation pressure is $P_s = 1.013\,bara$. No choke valve or gas injection was considered.

Figure 9 shows results of a simulation for $j_{g\,0} = 0.063\,m/s$ and $j_{l\,0} = 0.124\,m/s$ for representative variables: gas (j_{g1} and liquid (j_{l1}) superficial velocities at the bottom of the riser, position (x) of the liquid penetration front, void fraction (α_1) and pressure (P_1) at the bottom of the riser, liquid level at the riser (s_u) and void fraction (α_p) and pressure drop ($P_{pe} - P_{pt}$) in the stratified region at the pipeline.

It can be seen that, in this case, the stationary state used as the initial condition is not stable and the system goes to a limit cycle. It can be observed a large variation of superficial velocities, compared to the initial stationary values. Variations in the void fraction at the pipeline are relatively small, supporting the assumption of constant void fraction made in previous models. On the other hand, variations in pressure drop at the pipeline can be large compared to the stationary values, particularly in the blowdown stage.

Many parameters corresponding to the transient can be calculated from Fig. 9. Considering that the slugging cycle begins when the gas passage at the bottom of the riser is blocked, the severe slugging period and times corresponding to different stages described in Section 1. can be calculated. In this case, it can be seen that the liquid level does not remain at the top of the riser. From the simulations, it is also possible to determine the pressure amplitude at the bottom of the riser, the maximum position of the liquid penetration front in the pipeline and the minimum position of the liquid level in the riser.

Table 1 shows a comparison of experimental and model simulated severe slugging periods for different gas and liquid superficial velocities. The periods calculated with the model are in very good agreement with the experimental ones. There are some cases in which the simulation predicts a unstable condition while the experiment reports a stable condition.

With the simulations, it is also possible to obtain the numeric stability curve by keeping constant a value of liquid or gas flow rate and varying the other in fixed increments until passing from one condition (stable or unstable) to another; when this happens, the procedure is repeated with half the increment until achieving convergence. The procedure is laborious and computationally costly. Stability maps generated for catenary risers can be seen in (Baliño *et al.*, 2010) and (Nemoto and Baliño, 2012).

Figure 10 shows the numerically generated stability map for the conditions corresponding the experimental conditions of (Taitel *et al.*, 1990). In the same figure the experimental data points are shown; these unstable data points were classified as "unstable fall" ($s_u < s_t$ in the transient) or "unstable no fall" ($s_u = s_t$ always in the transient), based on a visual observation. It can be seen that the numerically generated stability curve includes all the unstable data points. Data points for which there is a discrepancy between experiment and stability simulation prediction are located close to the stability curve; for these points pressure amplitudes and liquid penetration lengths are small and liquid level in the riser remains at the top or close to it, making difficult to differentiate between the severe slugging instability condition and the fluctuations associated to the intermittent flow based only on a visualization. Besides, the experimental determination of the stability curve requires a very careful control of variables such as separator pressure and input flows; cases 5 and 7 in Table 1, for instance, show a large discrepancy in experimental period for almost the same values of superficial velocities, indicating that other variables were not kept constant in the experiment.

Table 1: Comparison with experimental results (Taitel *et al.*, 1990).

	Experiment			Simulation	
Case	$j_{g\,0}$ (m/s)	$j_{l\,0}$ (m/s)	T_{exp} (s)	T_{sim} (s)	error (%)
1	0.063	0.124	24	25.4	5.8
2	0.064	0.209	20	19.5	−2.5
3	0.123	0.183	15	15.5	3.3
4	0.124	0.212	14	14.6	4.3
5	0.062	0.679	6	7.36	22.7
6	0.063	0.367	13	13.3	2.3
7	0.063	0.679	9	7.35	−1.8
8	0.064	0.535	10	9.27	−7.3
9	0.065	0.226	19	18.7	−1.6
10	0.122	0.374	11	10.8	−1.8
11	0.123	0.621	8	7.36	−8.0
12	0.126	0.228	13	14.0	7.7
13	0.187	0.226	11	11.9	8.2
14	0.188	0.466	8	8.22	2.8
15	0.188	0.502	7	7.86	12.3
16	0.19	0.312	10	10.1	1.0
17	0.058	0.705	steady	7.19	NA
18	0.063	0.698	steady	7.21	NA
19	0.122	0.730	steady	6.45	NA
20	0.126	0.673	steady	14.0	NA
21	0.126	0.085	steady	18.2	NA
22	0.184	0.127	steady	4.24	NA
23	0.185	0.161	steady	12.3	NA
24	0.187	0.551	steady	7.36	NA
25	0.188	0.755	steady	steady	NA
26	0.19	0.685	steady	6.24	NA
27	0.313	0.433	steady	14.0	NA
28	0.314	0.347	steady	steady	NA
29	0.319	0.614	steady	steady	NA
30	0.321	0.744	steady	steady	NA
31	0.43	0.604	steady	steady	NA
32	0.433	0.701	steady	steady	NA

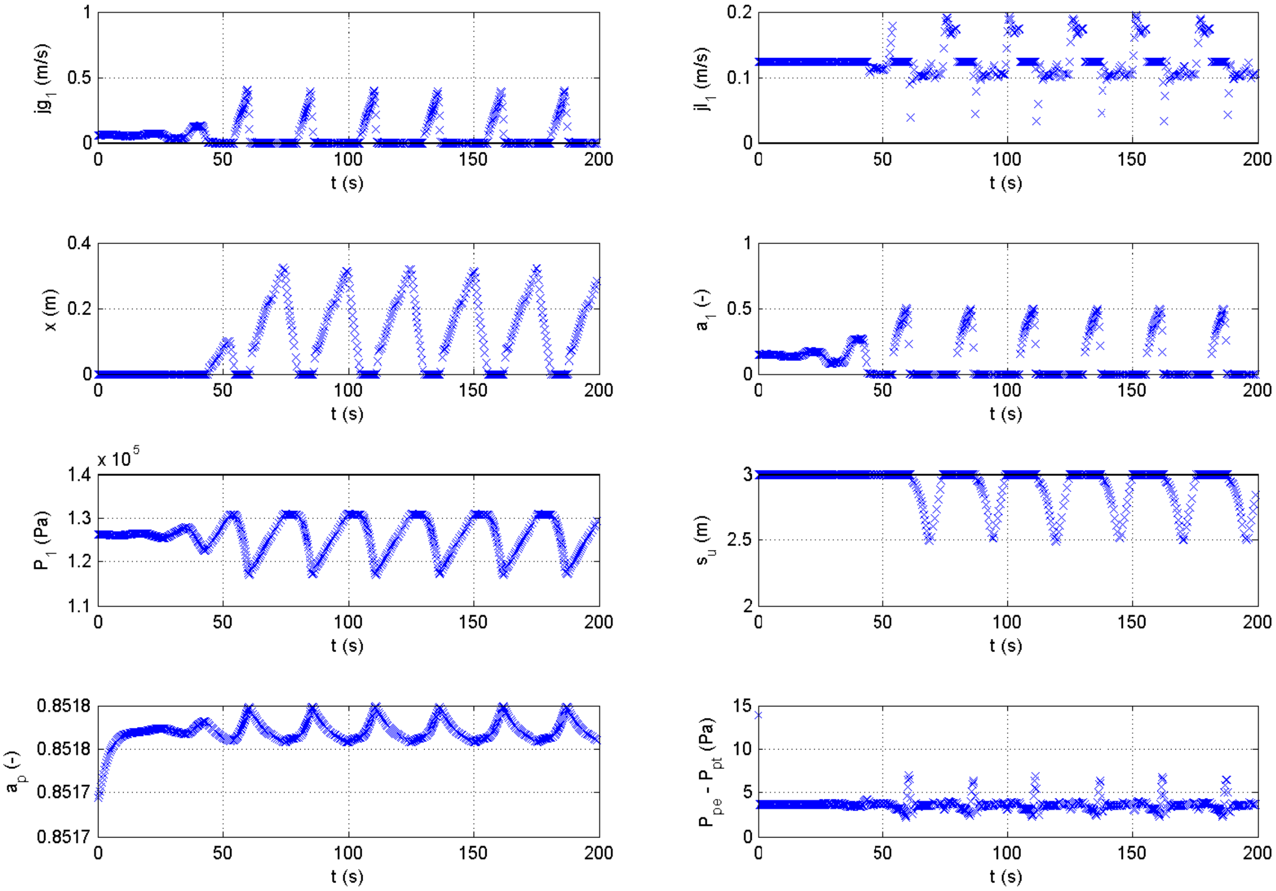


Figure 9: Simulation results for $j_{g\,0} = 0.063\,m/s$ and $j_{l\,0} = 0.124\,m/s$ (case 1, Table 1).

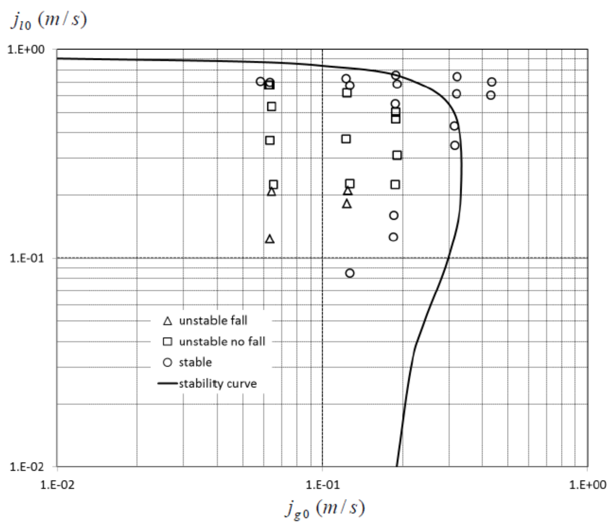


Figure 10: Numerically generated stability map and data from (Taitel *et al.*, 1990).

4.2 Data from (Jansen *et al.*, 1996)

In (Jansen *et al.*, 1996) it was used the same test facility as in (Taitel *et al.*, 1990). The simulations parameters were the same, except for $L_e = 10\,m$ and $\beta = 1\,^o$.

A choke valve with $C = 1.2 \times 10^5\,Pa\,s^2/m^2$, see Eq. (48), was introduced at the top of the riser in order to study the influence of choking on severe slugging; no gas injection was considered.

Table 2 shows a comparison of experimental and model simulated severe slugging periods for different gas and liquid superficial velocities. As in Section 4.1, the experimental and simulated periods are in very good agreement, with some cases with discrepancies in the prediction of the stability condition.

Table 2: Comparison with experimental results, choke valve, $C = 1.2 \times 10^5\,Pa\,s^2/m^2$ (Jansen *et al.*, 1996).

	Experiment			Simulation	
Case	$j_{g\,0}$ (m/s)	$j_{l\,0}$ (m/s)	T_{exp} (s)	T_{sim} (s)	error (%)
1	0.0753	0.0959	46.6	54.9	17.8
2	0.1147	0.0949	37.6	43.7	16.2
3	0.1739	0.0959	31.8	32.8	3.1
4	0.0781	0.0497	47.5	56.2	18.3
5	0.1181	0.0487	38.5	43.7	13.5
6	0.0809	0.1704	45	50.4	12.0
7	0.1209	0.1693	39.9	42.4	6.3
8	0.1713	0.0497	31.5	31.8	1.0
9	0.1209	0.2365	steady	43.0	NA
10	0.1734	0.2354	steady	36.7	NA
11	0.2493	0.2386	steady	30.6	NA
12	0.1698	0.1693	steady	35.5	NA
13	0.2474	0.1704	steady	28.5	NA
14	0.2502	0.0959	steady	25.0	NA
15	0.251	0.0497	steady	22.0	NA

In another experimental campaign, a constant gas superficial velocity at standard condition $j_{g0\,gl} =$

$0.091\,m/s$ was injected at the bottom of the riser in order to study the influence of gas lift on severe slugging; no choke valve was considered. The gas superficial velocity and the corresponding gas mass flow are related by:

$$j_{g0\,gl} = \frac{R_g\,T_0\,\dot{m}_{g\,l}}{P_0\,A} \tag{55}$$

Table 3 shows a comparison of experimental and model simulated severe slugging periods for different gas and liquid superficial velocities. Again, the agreement for experimental unstable data points is very good.

Table 3: Comparison with experimental results, gas injection, $j_{g0\,gl} = 0.091\,m/s$ (Jansen *et al.*, 1996).

	Experiment			Simulation	
Case	$j_{g\,0}$ (m/s)	$j_{l\,0}$ (m/s)	T_{exp} (s)	T_{sim} (s)	error (%)
1	0.0808	0.2528	21.5	29.8	38.6
2	0.1153	0.2549	18.9	23.8	25.9
3	0.1702	0.2571	14.9	18.1	21.5
4	0.2515	0.2582	13.4	13.2	−1.5
5	0.0791	0.152	24.1	36.9	53.1
6	0.1147	0.152	20.5	27.3	33.2
7	0.3125	0.1542	10.8	11.0	1.9
8	0.1695	0.1013	16.8	19.4	15.5
9	0.248	0.0949	13.2	13.9	5.3
10	0.1129	0.0487	27	28.0	3.7
11	0.1737	0.0476	18.8	19.7	4.8
12	0.3215	0.0916	9.5	11.1	16.8
13	0.2489	0.0465	14	13.6	−2.9
14	0.366	0.1552	steady	9.82	NA
15	0.4115	0.1552	steady	8.96	NA
16	0.369	0.0981	steady	9.41	NA
17	0.3141	0.0444	steady	10.4	NA

5. CONCLUSIONS

A mathematical model and numerical simulations corresponding to severe slugging in air-water pipeline-riser systems, are presented. The model is an improvement of the one previously published by the author (Baliño *et al.*, 2010), including inertial effects. Inertial effects are taken into account by using the rigid water-hammer approximation, which was numerically implemented without increasing substantially the complexity of the model.

A comparison is made with experimental results published in literature for vertical risers including the effect of a choke valve at the top and gas injection at the bottom of the riser, showing very good results for slugging cycles and stability maps.

ACKNOWLEDGEMENTS

This work was supported by *Petróleo Brasileiro S. A.* (Petrobras). The financial support of *Fundação de Amparo à Pesquisa do Estado de São Paulo* (FAPESP) to attend IMAACA 2012 is deeply acknowledged.

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