OBSERVER-BASED ACTUATOR AND SENSOR FAULT ESTIMATION

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ABSTRACT
A simple and straightforward unknown input observer with application to process fault estimation is presented. The observer information is devoted to the fault estimation for fault detection and isolation but it can be used to form an additional control input to accommodate the fault, since an estimated state vector is also obtained. An extension to fault sensor is discussed. The scheme is verified through simulation studies performed on the control of a vertical takeoff and landing aircraft in the vertical plane..

Keywords: actuator and sensor fault Estimation, unknown input observer, linear systems.

1. INTRODUCTION

Modern engineering systems need more reliable operating conditions because of increased productivity requirements. In order to improve reliability, an alarm occurs with a fault detection and isolation (FDI) scheme in the monitored system. The FDI problem is an attractive topic which has received considerable attention with different approaches.

Many FDI methods are observer based: the plant output is compared with an estimation provided by an observer, and a residual is calculated. The concept of unknown input observer (UIO) schemes were developed and in that case, the FDI scheme can detect, isolate and estimate faults. Some structural conditions are required (infinite zero structure and finite structure properties) on the model. The UIO problem can then be first studied and then applied on the particular FDI scheme.

Different approaches give solvability conditions and constructive solutions for the UIO problem. For LTI models, constructive solutions with reduced-order observers are first published with the geometric approach [15], [2], [1]. Constructive solutions based on generalized inverse matrices taking into account properties of invariant zeros are given in [21] and then in [22] and [17] with observability and detectability properties. Full order observers are then written in a similar way (based on generalized inverse matrices) in [8] and [7], but with some restriction on the infinite structure of the model. The algebraic approach is published in [29] and in [6] for continuous and discrete time systems, without restriction on the infinite structure of the model. New developments are now proposed with an observer based approach for some classes of nonlinear systems with a fuzzy approach [32], fuzzy systems with time delays [28] or with uncertain systems [4].

The structural invariants which play a fundamental role in this problem have been extensively studied in many papers and books [1], [23], [26], [19], [14], [20]. The knowledge of zeros is often an important issue because zeros are directly related to some stability conditions of the controlled system and the infinite structure is often related to solvability conditions.

The FDI problem with a fault diagnosis observer based approach has been developed in many papers. [30] developed an FDI observer by directly using the result of [21]. A systematic investigation with new design principles are written in [12] with some examples. Other developments are proposed in [18], [10] [31], [25] and [35]. Two survey papers are proposed in [11] and [13].

The objective of this paper is the development of an UIO for linear systems when there are two kinds of inputs: measured and unmeasured inputs with application to observer-based fault estimation. This UIO is proposed in a previous work [33] for disturbance estimation and rejection. This work makes a contribution by using this UIO for sensor fault estimation. The second section gives the description of this observer and its properties. Particularly, since the state equations of this observer are exactly the same as the initial model with an added term, it is proved to be accurate for simulation and for an integrated design approach. This kind of observer is usually dedicated to actuator fault detection; an extension to sensor fault detection is proposed. In the third part, the new scheme is verified through simulation studies performed on the control of a vertical takeoff and landing aircraft in the vertical plane.

2. UNKNOWN INPUT OBSERVER

Consider the linear system (1). $x \in \Re^n$ is the state vector, $z \in \Re^p$ is the vector set of measured variables and $y \in \Re^{p_1}$ is the vector of output variables to be controlled. The input variables are divided into two sets. $u \in \Re^m$ represents the known input vector and $d(t) \in \Re^q$ is the unknown input vector (disturbance or additive actuator fault), with $p \geq q$. Matrices A, B, F and H are supposed to be full rank matrices.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Fd(t) \\ z(t) = Hx(t) \\ y(t) = Cx(t) \end{cases} \qquad (1)$$

2.1 Preliminaries

Some assumptions are required for the state space model $\Sigma(H,A,F)$. The first one is non restrictive in a physical approach. These properties are developed in section 2.3.

Assumption 1. It is supposed that system $\Sigma(H,A,F)$ defined in (1) is controllable/observable and that the state matrix A is invertible.

Assumption 1 is not a restrictive condition for systems modeled with a physical approach, for example with a bond graph approach, because in that case, state variables are energy variables and except for particular cases, the eigenvalues of the state matrix are different from 0, and the state matrix is thus invertible. In a same way, the state model is controllable and observable. If the state matrix is non-invertible, a simple extension could be proposed.

Assumption 2. Matrix HF is a full column rank matrix.

If $p > q$, matrix HF is not a square matrix and matrix H is written as $H = [H_1^t H_2^t]^t$ with a reordering of measured output variables in order to have a full rank square matrix H_1F. If several choices are possible, the reordering will depend on the studied problem: actuator or sensor fault estimation. Similarly, if matrix HF is a not a full column rank matrix, a reduced-order state estimation can be proposed.

Assumption 3. The invariant zeros s of system $\Sigma(H,A,F)$ in (1) satisfy $Re(s) < 0$.

Necessary condition in assumption 2 for the existence of observers is often required (see [21]; [8]) and is called *observer matching condition*. It is also defined as an infinite structure requirement.

The condition on invariant zeros in assumption 3, or equivalently the strong detectability property defined in [16] corresponds to the minimum-phase condition, directly related to the zeros of system $\Sigma(H,A,F)$ (finite structure) defined as to be the values of $s \in \mathscr{C}$ (the complex plane) for which (2) is verified.

$$\mathtt{rank}\begin{pmatrix} sI-A & -F \\ H & 0 \end{pmatrix} < n + \mathtt{rank}\begin{pmatrix} -F \\ 0 \end{pmatrix} \quad (2)$$

2.2. Unknown Input Observer

The state equation (1) without output variable $y(t)$ is now rewritten as (3).

$$\begin{cases} x(t) = A^{-1}\dot{x}(t) - A^{-1}Bu(t) - A^{-1}Fd(t) \\ z_1(t) = H_1A^{-1}\dot{x}(t) - H_1A^{-1}Bu(t) - H_1A^{-1}Fd(t) \\ z_2(t) = H_2A^{-1}\dot{x}(t) - H_2A^{-1}Bu(t) - H_2A^{-1}Fd(t) \end{cases} \quad (3)$$

If matrix $H_1A^{-1}F$ is invertible (Model $\Sigma(H_1,A,F)$ has no null invariant zero), vector $d(t)$ can be written as in equation (4) and then the estimation of the disturbance variable can be written as in equation (5). The extension to models with a non-invertible matrix $H_1A^{-1}F$ is not proposed in this paper.

$$d(t) = -(H_1A^{-1}F)^{-1}[z_1(t) - H_1A^{-1}\dot{x}(t) + H_1A^{-1}Bu(t)] \quad (4)$$

$$\hat{d}(t) = -(H_1A^{-1}F)^{-1}[z_1(t) - H_1A^{-1}\dot{\hat{x}}(t) + H_1A^{-1}Bu(t)] \quad (5)$$

Since equation (6) is satisfied for the state vector, a new estimation is proposed for the state vector, defined in equation (7).

$$x(t) = A^{-1}\dot{x}(t) - A^{-1}Bu(t) - A^{-1}Fd(t) \quad (6)$$

$$\hat{x}(t) = A^{-1}\dot{\hat{x}}(t) - A^{-1}Bu(t) - A^{-1}F\hat{d}(t) + K(\dot{z}(t) - \dot{\hat{z}}(t)) \quad (7)$$

The state estimation equation deduced from (7) can also be written as (8), which is similar to a classical estimation, but with a difference in the last term. It needs the derivation of the measured variables. By Comparison with numerous existing UIO methods proposed in the literature, the advantage of this new approach is that the model of the observer (apart the derivative of the measurement) is exactly the model of the physical system. Note that it is not the case for other methods. It is a main advantage for example in an integrated design approach with physical considerations. Some discussions on the influence of noise measurement are proposed in the following. Note that in some often cited papers, [6], a r^{th} derivative (infinite zero order of the output variable) is needed for the output variable ($(r-1)^{th}$ for the input control variable) and that in [7], only state estimation with pseudo-inverse matrices is proposed. Note that if variables in vector $d(t)$ are associated to actuators faults, a FDI procedure with accommodation can be designed.

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + F\hat{d}(t) - AK(\dot{z}(t) - \dot{\hat{z}}(t)) \quad (8)$$

2.3. Properties of the observer

The convergence of the disturbance variables can be verified with equation (9), obtained from (4) and (5).

$$d(t) - \hat{d}(t) = (H_1A^{-1}F)^{-1}H_1A^{-1}(\dot{x}(t) - \dot{\hat{x}}(t)) \quad (9)$$

The estimation of the disturbance variables converges to the disturbance variable only if $(\dot{x}(t) - \dot{\hat{x}}(t))$ converges asymptotically.

In order to simplify notations, new matrices N_{BO} and N_{BF} are introduced in (10).

$$\begin{cases} N_{BO} = A^{-1} - A^{-1}F(H_1A^{-1}F)^{-1}H_1A^{-1} \\ N_{BF} = A^{-1} - A^{-1}F(H_1A^{-1}F)^{-1}H_1A^{-1} - KH \end{cases} \quad (10)$$

From (6) and (7), with $e(t) = x(t) - \hat{x}(t)$ it comes (11).

$$e(t) = N_{BF}\dot{e}(t) \quad (11)$$

Convergence of the state estimation must be proved with the study of the observer fixed poles. In equation (11), conditions for pole placement with matrix K are studied. If matrix N_{BF} is invertible, a classical pole placement is studied, and the error variable $e(t) = x(t) - \hat{x}(t)$ does not depend on the disturbance variable. The conditions for (8) to be an asymptotic state observer of $x(t)$ is that N_{BF} must be a Hurwitz matrix, i.e., has all its eigenvalues in the left-hand side of the complex plane.

Some properties of the observer are now explained. The proofs are in Appendix A. It is proved that this new observer must verify the matching condition defined in some well known approaches [16], [7] and that in that case, fixed poles of the estimation error are all the invariant zeros of system $\Sigma(H,A,F)$.

A necessary condition for the existence of the state estimator is proposed in proposition 1.

Property 1: A necessary condition for matrix N_{BF} defined in (10) to be invertible is that $\text{rank}(HF) = q$.

Condition defined in proposition 1 means that all the infinite zero orders of system $\Sigma(H,A,F)$ are equal to 1. This set of global infinite zero orders contains q integers.

It is now supposed that the condition $\text{rank}(HF) = q$ is satisfied. Two properties are proposed and proved in appendix.

Property 2: Matrix N_{BO} has q eigenvalues equal to 0.

Property 3: The fixed poles of the state estimation error defined in (11) are the invariant zeros of model $\Sigma(H,A,F)$

This property is proved with the study of the observability property of model $\Sigma(H,N_{BO})$.

2.4. Unknown Output observer

The method can be extended to the sensor observation or the sensor fault case with an augmented model. The system under consideration is written as (12), [9] and [35].

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) \\ z(t) = Hx(t) + Df_s(t) \end{array} \right. \tag{12}$$

$f_s(t)$ represents the sensors fault vector and D is a full column rank matrix. Consider a new state vector $x_s(t) \in \Re^p$ and a new state equation (13).

$$\dot{x}_s(t) = -A_s x_s(t) + A_s H x(t) + A_s D f_s(t) \ . \tag{13}$$

With a new state vector $\bar{x}(t) = (x^t(t) \ \ x_s^t(t))^t$, the augmented system can be expressed as (14).

$$\left\{ \begin{array}{l} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}u(t) + \bar{F}f_s(t) \\ \bar{z}(t) = \bar{H}\bar{x}(t) \end{array} \right. \tag{14}$$

The new matrices $\bar{A}$, $\bar{B}$, $\bar{H}$ and $\bar{F}$ are defined as follows:

$$\bar{A} = \begin{pmatrix} A & 0 \\ A_s H & -A_s \end{pmatrix} , \ \bar{B} = \begin{pmatrix} B \\ 0 \end{pmatrix} , \ \bar{F} = \begin{pmatrix} 0 \\ A_s D \end{pmatrix}$$

$$\bar{H} = \begin{pmatrix} 0 & I_p \end{pmatrix}$$

From the above augmented system, sensor fault may be treated as an actuator fault problem studied with an observer based approach and the properties of the new model can be easily pointed out. Remark that matrix D can be equal to the identity matrix (this choice is possible for physical systems).

Suppose first that only one observer is used for all sensors fault detection. In that case, A_s is chosen as a Hurwitz matrix with a good response time for variables x_s defined in equation (13), and $D = I_p$. Since $\bar{H}\bar{F}$ is a square invertible matrix, matrix $\bar{H}$ is not rewritten with two submatrices. In that case, some simple results are obtained:

$$\bar{A}^{-1} = \begin{pmatrix} A^{-1} & 0 \\ HA^{-1} & -A_s^{-1} \end{pmatrix} , \ -(\bar{H}\bar{A}^{-1}\bar{F}) = I_p$$

$$N_{BO} = \begin{pmatrix} A^{-1} & 0 \\ 0 & 0 \end{pmatrix} , \ N_{BF} = \begin{pmatrix} A^{-1} & K_1 \\ 0 & K_2 \end{pmatrix}$$

In that case, the set of invariant zeros is equal to the set of system poles (eigenvalues of the state matrix), which is also the set of fixed poles for the estimation error equation (11) for the extended system (14), because matrix N_{BF} is a block-diagonal matrix with one block equal to matrix A^{-1}. Due to time convergence of the estimation error compared with the system one, this observer must be modified in order to have appropriate fixed modes. Using one observer (or one sensor) is thus not successful in fault isolation problem.

A bank of observers is often proposed in the literature for actuator fault detection or isolation [5] and sensor fault isolation in presence of unknown disturbance or model uncertainties [3], [34].

New output vectors z_j and z^j can be defined with new output equations, equation (15)

$$\left\{ \begin{array}{l} z^j(t) = H^j\bar{x}(t) + f_s^j(t) \\ z_j(t) = h_j\bar{x}(t) + f_{sj}(t) \end{array} \right. \tag{15}$$

h_j is the j^{th} row of matrix H, H^j is obtained from matrix H by deleting the j^{th} row h_j. z_j is the j^{th} component of vector z and z^j is obtained from vector z by deleting the j^{th} variable z_j. The extended model (14) can be rewritten as equation (16). $\bar{F}_j$ is the j^{th} column of matrix $\bar{F}$ and $f_{sj}(t)$ is the j^{th} fault variable associated to the j^{th} sensor.

$$\left\{ \begin{array}{l} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}u(t) + \sum \bar{F}_j f_{sj}(t) \\ \bar{z}^j(t) = \bar{H}^j\bar{x}(t) \\ \bar{z}_j(t) = \bar{h}_j\bar{x}(t) \end{array} \right. \tag{16}$$

It is supposed that each subsystem has only one faulty sensor, in that case, p UIO can be constructed from the augmented system defined in equation (16) and only matrix $\bar{F}_j$ is used for the j^{th} observer. Equations defined in (3) are written with $z_1(t) = \bar{z}_j(t)$ and $z_2(t) = \bar{z}^j(t)$. The state vector estimates $\hat{\bar{x}}$ and the p unknown variables $f_{sj}(t)$ are calculated from equations (8) and (5) from the new state equations (16).

If one sensor is faulty and if the system keeps on at least two other sensors, it is possible to accommodate a new observer with safe sensors by a simple modification in vector $z_2(t) = \bar{z}^j(t)$. In that case new estimates must defined with new matrices for pole placement. This problem is not treated in this paper. It is also possible to use this new observer for actuator and sensor fault detection and isolation at the same time, but with same more conditions.

3. EXAMPLE

A VTOL (vertical takeoff and landing) aircraft in the vertical plane was studied by [24] and [27]. Its linearized dynamics are given in the state space formulation as (1), where

$$x = \begin{bmatrix} v_h - \text{horizontal velocity} \\ v_v - \text{vertical velocity} \\ q - \text{ pitch rate (deg/s)} \\ \theta - \text{pitch angle (deg)} \end{bmatrix}$$

$$u = \begin{bmatrix} \delta_c - \ \text{collective pitch control} \\ \delta_l - \ \text{longitudinal cyclic pitch control} \end{bmatrix} .$$

The two inputs are used to control the vertical motion and horizontal velocity of the aircraft, respectively. An unknown input (an actuator fault) is implemented in the system. The model parameters are given as follows

$$A = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = F = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ 5.52 & 4.49 \\ 0 & 0 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The system poles are located at $\{-6.8271, -2.5506\}$ and $\{-1.0112 \pm 1.5146j\}$, and there is no invariant zero in system $\Sigma(H, A, F)$.

In this paper, we do not consider measurement noises. A first order filter is used before the derivative of the measurement variable. Simulations (not proposed in this paper) can prove the robustness of the approach with some kind of noises. Theoretical developments for the study of the influence of noises on the convergence and on the applicability of the proposal for industry-sized examples will be proposed in an extended paper.

3.1. Actuator fault

The known input $u = [1\ 1]^t$ is implemented to the system with an initial condition of state variable $x_0 = [0.01\ 0\ 0\ 0]$ and initial conditions equal to zero for the observer variables. Since a fault actuator study is proposed, matrices B and F are equal and the unknown input is $d(t) = f(t)$ and $f(t) = [f_1(t)\ f_2(t)]^t$ is chosen as (17).

$$f_1(t) = \begin{cases} 0 & 0 \leq t \leq 2 \\ 0.3 & 2 < t \leq 3 \end{cases} \qquad f_2(t) = 0. \tag{17}$$

The existence conditions of an UIO are satisfied. Matrix HF is a full rank matrix and $\Sigma(H, A, F)$ has no invariant zero. Since matrix HF is not a square matrix, matrix H_1 in this example contains only the two first rows of matrix H, and $H_1 F$ is a square full rank matrix. In the state estimation equation defined in (8), matrix $K = [k_{ij}], i = 1, \ldots, 4, j = 1, \ldots, 3$ is used for pole placement. All the poles can be chosen, because fixed poles for the estimation problem are the invariant zeros. The four poles of matrix N_{BF}^{-1} defined in the state estimation error equation (11) are chosen as $s_1 = -20$, $s_2 = -25$, $s_3 = -\frac{100}{3}$ and $s_4 = -50$, compared with system's poles.

To validate the new results, different time responses of system variables are shown. A comparison between variables and their estimates is proposed. The estimated output variables $\{\hat{z}_1, \hat{z}_2, \hat{z}_3\}$ and the estimated unknown variables (actuators faults) $\{\hat{f}_1, \hat{f}_2\}$ are very close to the real variables, Fig. 1 and Fig. 2. With Fig. 3 it is proved that the estimation errors for the state variables converge rapidly to zero. From the above simulations, it can be concluded that asymptotic convergence of fault estimation can be achieved.

The robustness of this new observer is also proved with a time varying fault. In Fig. 4, a time varying fault is

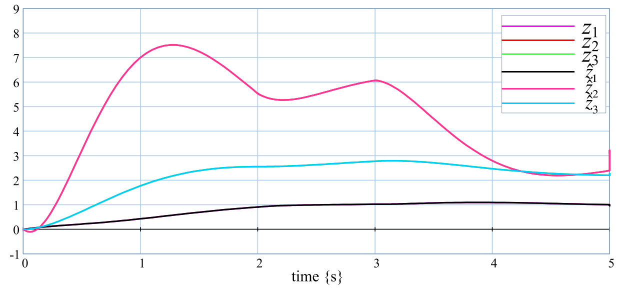

Figure 1: Output variable $z(t)$ and their estimates $\hat{z}$

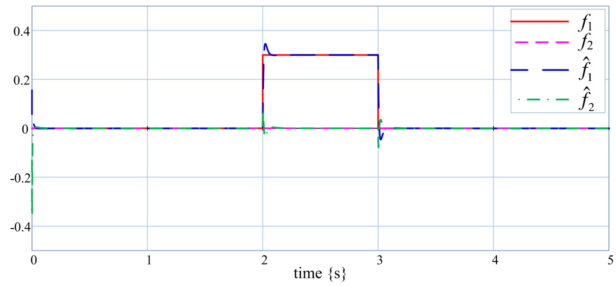

Figure 2: Fault variables $f(t)$ and their estimates $\hat{f}$

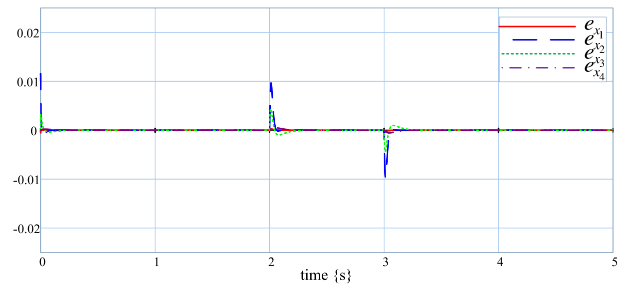

Figure3: Trajectories $e_i = x_i - \hat{x}_i, i = 1,\ldots,4$ with UIO in (8)

considered with a rather high frequency. The asymptotic convergence property and the good performance of the observer is proved in that case. It is also proved in case of a system with not well known parameters (not drawn here).

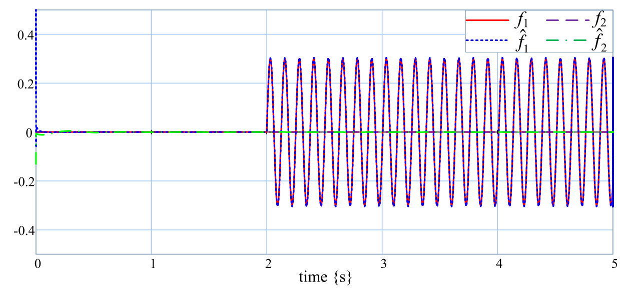

Figure 4: Fault variables $f(t)$ and their estimates $\hat{f}$: time varying fault

3.2. Sensor fault

The sensor fault analysis is now proposed on this example. It is supposed that the first sensor can be faulty. The new matrices are written in Appendix B. The extended model defined in (14) is now a 5^{th} order model. The new 5^{th} state variable is x_s and the new state matrix is chosen as $-A_s = -25$. The five poles of matrix N_{BF}^{-1} defined in the state estimation error equation (11) are located at $\{-30, -40, -40, -50, -50\}$. It can be shown that the extended model $\Sigma(\bar{H}, \bar{A}, \bar{F})$ has no invariant zero, and thus for this new estimation problem, there is not any fixed mode.

With a simple pole placement for matrix N_{BF}, it comes:

$$K = \begin{bmatrix} 0 & -0.0267 & -0.4457 \\ 0 & -0.6815 & 3.1666 \\ 0 & 0.0008 & 0.9954 \\ 0 & -0.0605 & -0.3685 \\ 0.0250 & 0 & 0 \end{bmatrix}$$

It is supposed that the faulty sensor is defined as a disturbance $d(t) = f_{s1}(t)$ which is a pulse signal with start time $1s$, end time $2s$ and amplitude 1. Variable $d(t)$ and its estimated are shown in Fig. 5.

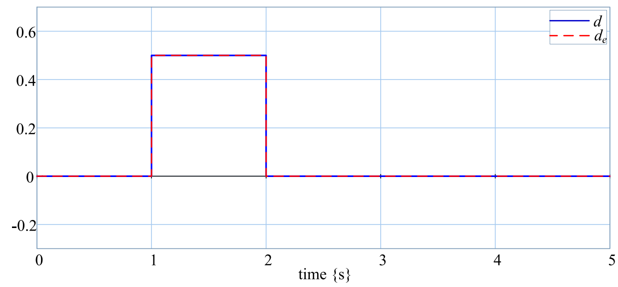

Figure 5: Fault variable $f_{S1}(t)$ and its estimates $\hat{f}_{S1}(t)$

Three variables are depicted in Fig. 6. The object is to compare the information given by the faulty sensor, its estimate and the "true" value of the system variable (horizontal velocity) obtained form the state estimation. In Fig. 6, time responses of variable $z_1(t)$ and its estimate $\hat{z}_1$ are very close. The third time response x_{1e} is the estimate of the first state variable and is the time response which should be obtained with a non faulty sensor. From these time responses, it is concluded that sensor fault is well solved and that fault accommodation is possible because a good estimate of the measured variable with a faulty sensor is obtained. Robustness issues could also be included, but due to lack of space, they are not proposed in this paper. By changing the plant model (parameters), it is shown by simulation that a good performance is obtained, and compared with some other well-known techniques, it is proved that this UIO is as well as other one.

Two other problems could be studied for this physical example: the case with one actuator fault and one sensor fault in the same model and secondly two sensors fault. The first situation is simple, the model must just be rewritten with the two kinds of fault. The second problem is simple if the two sensor faults don't occur simultaneously. Otherwise, only one sensor is no more sufficient for sensor fault estimation and accommodation.

4. CONCLUSION

An unknown input observer is proposed in this paper with application to the actuator and sensor fault detection and isolation problem. This observer is proved to be accurate with classical restrictive conditions based on the infinite structure and finite structure requirements. The application of this new scheme to a vertical takeoff and landing aircraft system shows that actuators and sensors faults can be estimated with satisfactory rapidity and accuracy. Theoretical developments for the study of the influence of noises on the convergence and on the applicability of the proposal for industry-sized examples will be proposed in an extended paper.

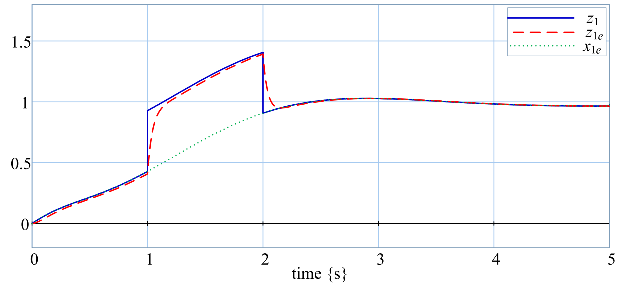

Fig. 6. Sensor time response $z_1(t)$, its estimate $\hat{z}_1(t)$ and state variable estimate $\hat{x}_1(t)$

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APPENDIX

Appendix A: Properties of the observer

Proof proposition 1

Matrix $N_{BF}F$ is equal to $[A^{-1} - A^{-1}F(H_1A^{-1}F)^{-1}H_1A^{-1} - KH]F$, thus $N_{BF}F = A^{-1}F - A^{-1}F(H_1A^{-1}F)^{-1}H_1A^{-1}F - KHF = KHF$. If the rank condition $\text{rank}(HF) = q$ is not satisfied, the Kernel of matrix N_{BF} is not empty, which means that matrix N_{BF} is not invertible and that this matrix contains at least one null mode, thus pole placement is not possible (all its eigenvalues are not in the left-hand side of the complex plane).□

Proof proposition 2

Since $H_1N_{BO} = H_1A^{-1} - H_1A^{-1}F(H_1A^{-1}F)^{-1}H_1A^{-1}$, it comes $H_1N_{BO} = 0$. Since vector H_1^t is orthogonal to matrix N_{BO}, matrix N_{BO} contains at least q null eigenvalues.

Proof proposition 3

First, the observability property of model $\Sigma(H, N_{BO})$ is studied. The non observable poles are the roots of the invariant polynomials obtained from the Smith form of matrix $N(s)$ defined in (18). With matrix H_1, only the q null modes of matrix N_{BO} can be assigned. The goal is to emphasize the number of modes which can be assigned with matrix H_2.

$$N(s) = \begin{pmatrix} sI - N_{BO} \\ H_1 \\ H_2 \end{pmatrix} \quad (18)$$

The fixed poles of the state estimation error defined in (11) are thus the non observable poles of model $\Sigma(H, N_{BO})$. Now, some equivalent transformations are proposed for the Smith matrix $S(s)$ of system $\Sigma(H, A, F)$ defined in (19).

$$S(s) = \begin{pmatrix} sI - A & -F \\ H_1 & 0 \\ H_2 & 0 \end{pmatrix} \quad (19)$$

$$S(s) \sim \begin{pmatrix} sA^{-1} - I & -A^{-1}F \\ H_1 & 0 \\ H_2 & 0 \end{pmatrix} \quad (20)$$

$$\sim \begin{pmatrix} sA^{-1} - I & -A^{-1}F \\ H_1 + sH_1A^{-1} - H_1 & -H_1A^{-1}F \\ H_2 & 0 \end{pmatrix} \quad (21)$$

$$\sim \begin{pmatrix} sA^{-1} - I & A^{-1}F(H_1A^{-1}F)^{-1} \\ sH_1A^{-1} & I \\ H_2 & 0 \end{pmatrix} \quad (22)$$

$$\sim \begin{pmatrix} sA^{-1} - I - A^{-1}F(H_1A^{-1}F)^{-1} - sH_1A^{-1} & 0 \\ sH_1A^{-1} & I \\ H_2 & 0 \end{pmatrix} \quad (23)$$

$$\sim \begin{pmatrix} sN_{BO} - I & 0 \\ 0 & I \\ H_2 & 0 \end{pmatrix} \sim \begin{pmatrix} sN_{BO} - I & 0 \\ H_2 & 0 \\ 0 & I \end{pmatrix} \quad (24)$$

The non observable modes are thus all the inverse of the invariant zeros of system $\Sigma(H, A, F)$. They are the fixed modes of the state estimation error equation.

Appendix B: Sensor fault model

$$\bar{A} = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 & 0 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 & 0 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 25 & 0 & 0 & 0 & -25 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ 5.52 & 4.49 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \bar{F} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 25 \end{bmatrix}$$

$$\bar{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\bar{H_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \bar{H_2} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$