Unknown Input Observer for MIMO Systems with Stability

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Abstract

This paper presents a solution for the state and unknown input estimator of linear MIMO systems with a structural approach. Compared to the classic Input-Output Decoupling problem, a systematic procedure is initially proposed for the analysis stage by analysing the finite and infinite structures of the modelled system from a structural approach. Afterwards, the observer is directly implemented from the original model at the synthesis stage in a graphical approach with some added terms, and can be represented by a Bond Graph model. The observer is tested by a particular and illustrative example which considers a real torsion-bar model.

Keywords: Unknown Input Observer, Bond Graph, Structural Analysis, MIMO Systems

1 Introduction

The unknown input estimation and state observability problem (UIO) is a well known problem. Different approaches give solvability conditions and constructive solutions for this problem.

At the analysis stage, before design, most of the approaches require the analysis of the structural invariants of the model. The knowledge of zeros (finite structure) is an important issue because these zeros are directly related to stability conditions of the observer ((Hautus 1983)) and of the controlled system. The infinite structure of the model is related to solvability conditions (see Appendix A).

For LTI models, constructive solutions with reduced order observers are first proposed with the geometric approach, see (Bhattacharyya 1978), (Basile and Marro 1973) or based on generalized inverse matrices like in (Miller and Mukunden 1982) and (Hou and Muller 1992). Full order observers are then proposed in a similar way (based on generalized inverse matrices), see (Darouach 2009) for some works related with this issue. Other approaches based on canonical forms, the algebraic approach or sliding mode observers are not recalled here.

This paper proposes an extension of some works dedicated to the UIO problem when the model contains some non strictly stable invariant zeros. The contribution consists on the definition of a new estimation of the disturbance variables which takes into account some integrals of the measured variables in order to augment the number of assigned poles.

2 Unknown Input Observer

Consider a linear perturbed system described by (1), where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^p$ is the vector of measurable variables. Vector $u \in \mathbb{R}^m$ represents the known input variables, whereas $d(t) \in \mathbb{R}^q$ is the vector which represents the unknown input variables. A, B, F, C are known constant matrices of appropriate dimensions.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Fd(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

Generally, the state vector $x(t)$ cannot be entirely measured and the system is often subject to unknown inputs $d(t)$ (disturbance or failure...) which must be estimated.

2.1 MIMO Systems

In this section, the UIO estimation for MIMO systems without null invariant zero is recalled. For some works related with the design of UIO for linear Bond Graph models, see (Yang et al. 2013), (Tarasov et al. 2013).

The UIO problem is recalled for the multi-variable case with two unknown input variables ($q = 2$) and two measured outputs variables ($p = 2$). It can easily be extended to any square model with $p = q$. It is supposed that system $\Sigma(C, A, F)$ is controllable, observable and invertible. The equation (1) can be written as (2).

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$$\begin{cases} x(t) = A^{-1}\dot{x}(t) - A^{-1}Bu(t) - A^{-1}Fd(t) \\ y(t) = CA^{-1}\dot{x}(t) - CA^{-1}Bu(t) - CA^{-1}Fd(t) \end{cases} \tag{2}$$

The matrix $\Omega_d = CA^{-1}F$ is invertible if model $\Sigma(C,A,F)$ has no null invariant zeros. Matrix Ω_d is similar to decoupling matrix Ω in control theory. In the classical input-output decoupling problem, the decoupling matrix Ω defined in equation (8) is used with matrix B instead of matrix F (with the control input variables). From (2), the disturbance vector $d(t)$ and its estimation $\hat{d}(t)$ are written in equation (3), and the disturbance equation error in equation (4). The estimation of the state vector is written in equation (5).

$$\begin{cases} d(t) = -\Omega_d^{-1}[y(t) - CA^{-1}\dot{x}(t) + CA^{-1}Bu(t)] \\ \\ \hat{d}(t) = -\Omega_d^{-1}[y(t) - CA^{-1}\dot{\hat{x}}(t) + CA^{-1}Bu(t)] \end{cases} \tag{3}$$

$$d(t) - \hat{d}(t) = \Omega_d^{-1}CA^{-1}(\dot{x}(t) - \dot{\hat{x}}(t)) \tag{4}$$

$$\begin{aligned} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + F\hat{d}(t) - \cdots \\ -AK \begin{bmatrix} y_1^{(n_1)}(t) - \hat{y}_1^{(n_1)}(t) \\ y_2^{(n_2)}(t) - \hat{y}_2^{(n_2)}(t) \end{bmatrix} \end{aligned} \tag{5}$$

Ω_d is defined in case of the Bond Graph representation with a derivative causality assignment (BGD) and related to the infinite structure of the BGD.

Consider vector $e(t)$ defined as $e(t) = x(t) - \hat{x}(t)$. It has been proved in (Tarasov et al. 2013) that vector $e(t)$ verifies equation (6) with matrix N_{CL} defined in (7). In that case, the state estimation error doesn't depend on the disturbance variable.

$$e(t) = N_{CL}\dot{e}(t) \tag{6}$$

$$\begin{cases} N_{OL} = A^{-1} - A^{-1}F\Omega_d^{-1}CA^{-1} \\ \\ N_{CL} = A^{-1} - A^{-1}F\Omega_d^{-1}CA^{-1} - K\begin{bmatrix} C_1A^{n_1-1} \\ C_2A^{n_2-1} \end{bmatrix} \end{cases} \tag{7}$$

If matrix N_{CL} is invertible, a classical pole placement is studied with matrix K used for pole placement. A necessary condition for the existence of the state estimator is proposed in Proposition 1.

Suppose that $\{n_1, n_2\}$ and $\{n'_1, n'_2\}$ are the set of row infinite zero orders and global infinite zero orders respectively, of system $\Sigma(C,A,F)$ (see Appendix A). Matrix Ω is necessary in this estimation problem.

$$\Omega = \begin{bmatrix} C_1A^{n_1-1}F \\ C_2A^{n_2-1}F \end{bmatrix} \tag{8}$$

Proposition 1 *A necessary condition for matrix N_{CL} to be invertible is that matrix Ω is invertible.*

Proof 1 *See appendix B*

It has been proved in (Tarasov et al. 2013) that the eigenvalues of matrix N_{OL} defined in (7) are the inverse of the invariant zeros of system $\Sigma(C,A,F)$ ($n - (n_1 + n_2)$) modes) plus $n_1 + n_2$ eigenvalues equal to 0. The null eigenvalues can be assigned to new values and the invariant zeros are the fixed poles as recalled in the two following propositions.

Proposition 2 *((Tarasov et al. 2013))In matrix N_{CL} defined in (7), $n_1 + n_2$ poles can be chosen with matrix K.*

Proposition 3 *((Tarasov et al. 2013)) The fixed poles of the estimation equation error defined in (6) are the invariant zeros of system $\Sigma(C,A,F)$.*

The proposed approach can then be applied for systems with *strictly stable invariant zeros*.

If the state equation (1) is written from a Bond Graph model, it is possible to design a Bond Graph model for the state estimation defined in (5) because the equation (5) is very close to the initial state equation. Some signal bonds must be added for the disturbance equation defined in (3). The structure of the observer is proposed in Fig.(1), where BG_{sys} is Bond Graph model of the system and BG_{obs} is for the observer Bond Graph model. A simplified block diagram in Fig.1 represents the structure of simulation to estimate unknown variables. As written in equation (5), output variables must be derived. Some remarks about this feature are made in the following.

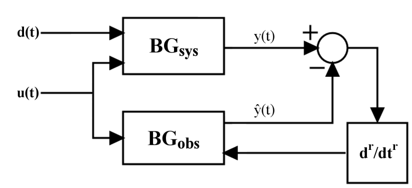

Figure 1: Structure of the simulation to estimate variables.

2.2 Simplified Case with Null Invariant Zeros

In this section, an extension of UIO observer is proposed when the model contains null invariant zeros. In this paper, we consider the case with one null invariant zero, it can be easily extended to more general situations.

The equation (1) can be written as (9)

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$$\begin{cases} x(t) = A^{-1}\dot{x}(t) - A^{-1}Bu(t) - A^{-1}Fd(t) \\ y_1(t) = C_1[A^{-1}\dot{x}(t) - A^{-1}Bu(t) - A^{-1}Fd(t)] \\ y_2(t) = C_2[A^{-1}\dot{x}(t) - A^{-1}Bu(t) - A^{-1}Fd(t)] \end{cases} \tag{9}$$

Consider, without restriction, that $C_1A^{-1}F = 0$. In that case, in the BGD, the causal path length between the output detector and the two disturbance inputs is at least equal to 1 (it is supposed to be equal to 1 in order to simplify the theoretical development). In (9), the mathematical expression of $y_1(t)$ and of its primitive is then

$$\begin{cases} y_1(t) = C_1A^{-1}\dot{x}(t) - C_1A^{-1}Bu(t) \\ \int y_1(t)\mathrm{d}t = C_1A^{-1}x(t) - C_1A^{-1}B\int u(t)\mathrm{d}t \end{cases} \tag{10}$$

Thus

$$\int y_1(t)\mathrm{d}t = C_1A^{-2}\dot{x}(t) - C_1A^{-2}Bu(t) - \cdots \\ - C_1A^{-1}B\int u(t)\mathrm{d}t - C_1A^{-2}Fd(t) \tag{11}$$

If model $\Sigma(C,A,F)$ has only one null invariant zero, matrix $C_1A^{-2}F \neq 0$ and matrix $\Omega_d = [(C_1A^{-2}F)^t, (C_2A^{-1}F)^t]^t$ is invertible. A new expression of vector $d(t)$ can be written, as well for $\hat{d}(t)$ from equation (11) in the same manner as in the classical case and the error equation is written in (14).

$$d(t) = -\Omega_d^{-1}\left[\begin{array}{l} \int y_1(t)\mathrm{d}t - C_1A^{-2}\dot{x}(t) + \gamma(u) \\ y_2(t) - C_2A^{-1}(\dot{x}(t) - Bu(t)) \end{array}\right] \tag{12}$$

$$\hat{d}(t) = -\Omega_d^{-1}\left[\begin{array}{l} \int y_1(t)\text{text}t - C_1A^{-2}\dot{\hat{x}}(t) + \gamma(u) \\ y_2(t) - C_2A^{-1}(\dot{\hat{x}}(t) - Bu(t)) \end{array}\right] \tag{13}$$

Where $\gamma(u) = C_1A^{-2}Bu(t) + C_1A^{-1}B\int u(t)\mathrm{d}t$, and then

$$d(t) - \hat{d}(t) = \Omega_d^{-1}\begin{bmatrix} C_1A^{-2} \\ C_2A^{-1} \end{bmatrix}(\dot{x}(t) - \dot{\hat{x}}(t)) \tag{14}$$

The estimation of the state vector $x(t)$ is the same as in equation (17) as well as for the state estimation error equation defined in (6). Nevertheless, expressions of matrices N_{OL} and N_{CL} have changed since the model has one null invariant zero and thus matrix N_{OL} contains $(n_1 + n_2 + 1)$ null eigenvalues. New expressions are written in (15)

$$\begin{cases} N_{OL} = A^{-1} - A^{-1}F\Omega_d^{-1} \begin{bmatrix} C_1A^{-2} \\ C_2A^{-1} \end{bmatrix} \\ N_{CL} = A^{-1} - A^{-1}F\Omega_d^{-1} \begin{bmatrix} C_1A^{-2} \\ C_2A^{-1} \end{bmatrix} \cdots \\ \qquad -K\begin{bmatrix} C_1A^{n_1-1} \\ C_2A^{n_2-1} \end{bmatrix} \end{cases} \tag{15}$$

A new proposition can be written.

Proposition 4 *The fixed poles of the estimation equation error defined in (15) are the strictly stable invariant zeros of system $\Sigma(C,A,F)$. $(n - (n_1 + n_2 + 1))$ fixed poles.*

Proof 2 *See appendix C*

As said before, this approach can be easily extended to MIMO models with several null invariant zeros. The idea consists in applying integration on the output variables. It is applied to the torsion-bar system.

2.3 Output Differentiation and noise

Numerical differentiation of measurable signals is a classical problem in signal processing and automation, and many problems have been solved by creating algorithms for approximation of derivatives. The numerical methods for approximation of derivatives of measurable signals can be used to obtain signals which are not known through measurements and reconstruct the missing system data.

There exist many ways in the literature to derivate signals. Some common features are the precision between derivative estimation and noise sensibility and perturbations. These noises or perturbations are the principal trouble for developing derivation algorithms. Most of them assume some features of signal derived and noise (perturbation) of this derivation.

Different approaches are used for different situations such as Linear Systems in (Luenberger 1971), (Carlsson, Ahlen, and Sternad 1991), (Diop et al. 1994), (Al-Alaoui 1993), (Dabroom and Khalil 1997), (Levant 1998), (Levant 2003), (Mehdi 2010). These approaches can be classified by two principal classes: *a*) Model Approach or *b*) Signal Approach. In this case of study, the simulations are performed using the *Matlab* and *20-SIM* softwares. Therefore, for a numerical differentiation of the output signal in simulations, will be used some own blocks of these softwares which are called direct derivative with a noise-filter inside. In future works will be used others approaches implemented over the real bar system.

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3 Torsion-bar System

3.1 Experimental System description

The experimental setup in the Fig. 2 is the real torsion bar system with its main parts.

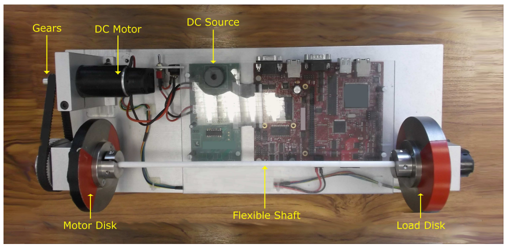

Figure 2: Real torsion bar system.

A functional schematic model of the torsion bar system is presented in Fig.3. According to Fig.3, the system consists of the following components: a DC Power Source, a classical DC Motor which is modelled by an electrical part (Inductance L_a and Resistance R_a) and a mechanical part (Inertia J_m is supposed negligible), a transmission element which transfer the rotation from the motor to the motor disk with a transmission ratio (k_{belt}), a first rotational disk (Motor Disk) with an inertial parameter J_1 and a friction coefficient R_1, a flexible shaft modelled as a spring-damper element (Spring c and Damper R_{shaft}), and a second rotational disk (Load Disk) with an inertial parameter J_2 and a friction coefficient R_2.

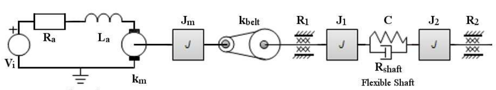

Figure 3: Schematic model of the real torsion bar system.

The simplified Bond Graph model of the system is shown in the Fig.4.

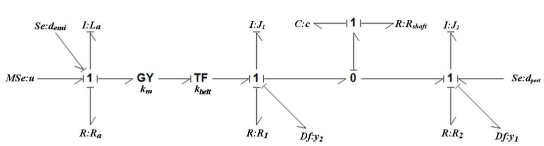

Figure 4: Simplified Bond Graph of the torsion bar system.

For the experimental system in Fig.4, the controlled input voltage is represented by a modulated effort source $MSe : u$. Moreover, y_1 and y_2 are speed rotational variables represented in the bond graph model by flow output detectors $Df : y_1$ and $Df : y_2$ respectively. These sensors are used to estimate the state variables and the unknown inputs $d(t)$ modeled by the source $Se : d_{pert}$ and d_{emi} modeled by the source $Se : d_{emi}$, which represents a torque applied to the first rotational disk(J_1) and a electromagnetic interference respectively. The numerical values for each element of the system are given in Table 1.

Table 1: Parameters for the experimental system

Element	Symbol	Value
Inductance	L_a	$0.34 \cdot 10^{-3}\ H$
Inertia of motor disk	J_1	$9.07 \cdot 10^{-4}\ kg \cdot m^2$
Inertia of load disk	J_2	$1.37 \cdot 10^{-3}\ kg \cdot m^2$
Spring compliance	C	$0.543\ N \cdot m/rad$
Resistance	R_a	$1.23\ \Omega$
Motor disk friction	R_1	$0.005\ N \cdot m \cdot s/rad$
Load disk friction	R_2	$25 \cdot 10^{-6}\ N \cdot m \cdot s/rad$
Damping spring	R_{shaft}	$5 \cdot 10^{-4}\ N \cdot s/rad$
Motor constant	k	$38.9 \cdot 10^{-3}\ N \cdot m/A$
Transmission ratio	k_{belt}	3.75

According to the Bond Graph model, a state space representation is performed as described in the form (1). The state vector $x = [x_1, x_2, x_3, x_4]^t$, is energy storage variables: $x_1 = q_c = q_{c_{shaft}}$ (angular displacement), $x_2 = p_{J_2}$, $x_3 = p_{J_1}$ (angular momentums), and $x_4 = p_{L_a}$ (flux linkage). The output matrix C can be written as $C = [{C_1}^t, {C_2}^t]^t$. The state equations are written as (16). The poles of the model (eigenvalues of matrix A) are equal to -3617.5, $-2.15 \pm 58j$, -2.2523.

$$\begin{cases} \dot{x}_1 = -\frac{1}{J_2}x_2 + \frac{1}{J_1}x_3 \\ \dot{x}_2 = \frac{1}{C}x_1 + a_{2,2}x_2 + \frac{R_{Shaft}}{J_1}x_3 + d_{pert} \\ \dot{x}_3 = -\frac{1}{C}x_1 + \frac{R_{Shaft}}{J_2}x_2 + a_{3,3}x_3 + \frac{k}{L_a.k_{belt}}x_4 \\ \dot{x}_4 = -\frac{k}{J_1 \cdot k_{belt}}x_3 - \frac{R_a}{L_a}x_4 + u + d_{emi} \\ \quad a_{2,2} = (-\frac{R_2}{J_2} - \frac{R_{Shaft}}{J_2})\ ;\ a_{3,3} = (-\frac{R_1}{J_1} - \frac{R_{Shaft}}{J_1}) \\ \quad y_1 = \frac{1}{J_2}x_2\ ;\ y_2 = \frac{1}{J_1}x_3 \end{cases} \tag{16}$$

3.2 Structural Analysis

From the Bond Graph model of Fig.4 the causal path between the output variable y_1 and the disturbance input d_{pert} is $Df : y_1 \to I : J_2 \to Se : d_{pert}$. the length of the causal path is equal to 1, then $n_1 = 1$. The causal path between the output y_2 and the disturbance input d_{emi} is $Df : y_2 \to I : J_1 \to TF : k_{belt} \to GY : k_m \to I : L_a \to Se : d_{emi}$. The length of the causal path is equal to 2. Another causal path between the second output detector and the set of unknown inputs is $Df : y_2 \to I : J_1 \to R : R_{shaft} \to I : J_2 \to Se : d_{pert}$. The length of the causal path is equal to 2, then $n_2 = 2$ and ma-

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trix Ω defined in equation (8) is invertible. With the infinite structure, the state estimation is written as equation (17).

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + F\hat{d}(t) - \cdots \\ - AK \begin{pmatrix} {y_1}^{(1)}(t) - \hat{y_1}^{(1)}(t) \\ {y_2}^{(2)}(t) - \hat{y_2}^{(2)}(t) \end{pmatrix} \quad (17)$$

Null invariant zeros are studied with the bond graph model with a derivative causality assignment, drawn in Fig.5.

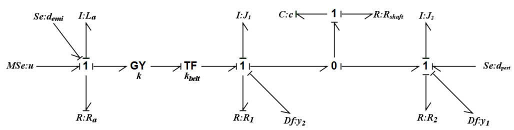

Figure 5: Bond Graph model with derivative causality.

In the BGD, the causal path between the output variable y_1 and the disturbance input d_{pert} is $Df : y_1 \rightarrow R : R_1 \rightarrow Se : d_{pert}$. The length of the causal path is equal to 0, then $n_{d_1} = 0$. The causal path between the output variable y_2 and the disturbance input d_{emi} is $Df : y_2 \rightarrow R : R_1 \rightarrow TF : k_{belt} \rightarrow GY : k_m \rightarrow R : R_a \rightarrow Se : d_{emi}$. The length of the causal path is equal to 0, then $n_{d_2} = 0$. These two paths are however not disjoint. Thus this model is not invertible for the BGD (infinite structure defined for the BGD) and matrix Ω_d is not invertible. There is a null invariant zero and the previous UIO cannot be used for this example. A solution to construct the UIO is presented below.

4 Unknown Input Observer: Torsion-Bar Application

4.1 Analysis

The torsion bar model has one null invariant zero because matrix $CA^{-1}F$ is not invertible. In that case neither matrix $C_1A^{-1}F$ nor $C_2A^{-1}F$ is equal to zero. The previous methodology cannot be directly applied. Nevertheless, it is quite similar because matrices $C_1A^{-1}F$ and $C_2A^{-1}F$ are proportional and linear combination between variables $y_1(t)$ and $y_2(t)$ can be applied in order to obtain a null row in matrix $CA^{-1}F$ before applying an integration. The procedure is now applied.

From the causal paths gains of the BGD model, or directly from matrix $CA^{-1}F$, it can be shown that the two rows are equal and then $y_1(t) - y_2(t)$ is written as in (18).

$$y_1(t) - y_2(t) = y_{1,2}(t) = (C_{1-2})A^{-1}\dot{x}(t) - \cdots \\ - (C_{1-2})A^{-1}Bu(t) \quad (18)$$

Where $C_{1-2} = (C_1 - C_2)$.
The primitive of variable $y_{1,2}(t)$ is thus (19)

$$\int y_{1,2}(t)\mathrm{d}t = y^*(t) = (C_{1-2})A^{-2}\dot{x}(t) - \cdots \\ - (C_{1-2})A^{-2}Bu(t) - (C_{1-2})A^{-2}Fd(t) - \cdots \\ - (C_{1-2})A^{-1}B \int u(t)\mathrm{d}t \quad (19)$$

We consider now two equations with $y_1(t)$ and $y^*(t)$ in (20). Note that it is possible to choose $y_2(t)$ and $y^*(t)$ as well. In that case, matrix Ω_d^* defined as $[(C_1A^{-1}F)^t, ((C_{1-2})A^{-2}F)^t]^t$ is invertible.

$$\begin{cases} y_1(t) = C_1[A^{-1}\dot{x}(t) - A^{-1}Bu(t) & -A^{-1}Fd(t)] \\ y^*(t) = (C_{1-2})[A^{-2}\dot{x}(t) - A^{-2} & Bu(t)\cdots \\ \quad -A^{-2}Fd(t) - A^{-1} & B\int u(t)\mathrm{d}t] \end{cases} \quad (20)$$

According to (19), the disturbance equation error (4) can be rewritten as (21).

$$d(t) - \hat{d}(t) = [\Omega_d^*]^{-1} \begin{bmatrix} C_1A^{-1} \\ (C_{1-2})A^{-2} \end{bmatrix} (\dot{x}(t) - \dot{\hat{x}}(t)) \quad (21)$$

Therefore, N_{LC} for this system is rewritten as (22), and $n_1 + n_2 + 1 = 4$ which indicates that the four poles can be assigned.

$$N_{CL} = A^{-1} - A^{-1}F[\Omega_d^*]^{-1} \begin{bmatrix} C_1A^{-1} \\ (C_{1-2})A^{-2} \end{bmatrix} - K \begin{bmatrix} C_1 \\ C_2A \end{bmatrix} \quad (22)$$

4.2 Simulation

The estimation of the unknown input variables is defined in (23). For pole placement the matrix K is obtained (24). The four poles are chosen as $-1/2000, -1/2100, -1/2200$ and $-1/2200$ for the matrix N_{CL}, but they are the inverse of the classical estimation error equation.

$$\hat{d}(t) = -[\Omega_d^*]^{-1} \left[\begin{bmatrix} y_1 \\ y^* \end{bmatrix} \begin{bmatrix} C_1A^{-1} \\ (C_{1-2})A^{-2} \end{bmatrix} \dot{\hat{x}}(t) + \cdots \right. \\ \left. + \begin{bmatrix} C_1A^{-1}B \\ (C_{1-2})A^{-2}B \end{bmatrix} u(t) \right] \quad (23)$$

$$K = \begin{bmatrix} -5.96 \cdot 10^{-11} & 1.08 \cdot 10^{-10} \\ 6.227 \cdot 10^{-7} & 8.8 \cdot 10^{-24} \\ -3.41 \cdot 10^{-10} & 6.18 \cdot 10^{-10} \\ -2.352 \cdot 10^{-8} & 4.266 \cdot 10^{-8} \end{bmatrix} \quad (24)$$

Different variables are simulated in *20-sim*. The input is chosen as $u(t) = 1v$(step response), unknown inputs are

$d_{emi} = 0.1A$ with action between 1 to 2sec. and $d_{pert} = 0.01N.m$ with time action between 3 to 4 sec.

First, the signal d_{pert} and its estimation $\hat{d}_{pert}$ are displayed in Fig.6. In Fig.7 is possible to see a zoom that shows more closely the behavior of the signals d_{pert} and $\hat{d}_{pert}$.

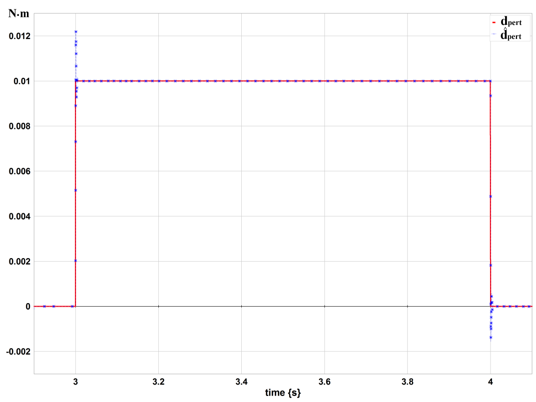

Figure 6: Unknown input d_{pert} and its estimation.

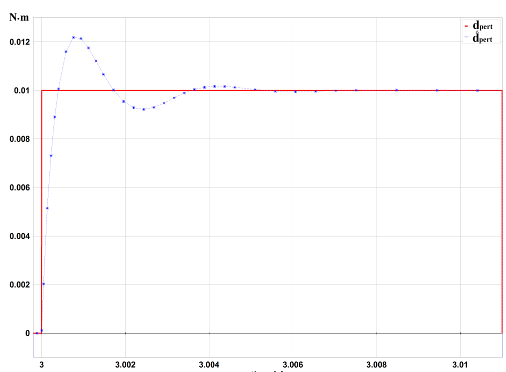

Figure 7: Zoom of unknown input d_{pert} and its estimation.

The signal d_{emi} and its estimation are shown in Fig.8, and a zoom of the signals is displayed in the Fig.9. It is possible to see the estimator reaction in this zoom.

The Fig.10a shows the output signal y_1 and its estimation $\hat{y}_1$ is displayed in Fig.10b.

Finally, the Fig.11 represents the output signal y_1 in (a) and the estimation $\hat{y}_2$ in (b).

These results show that the outputs estimations are well obtained and the estimations for disturbances have a quickly and very close responses at the disturbances.

5 Conclusion

An effective performance estimation system is essential in any controlled system subject to unknown inputs, failures

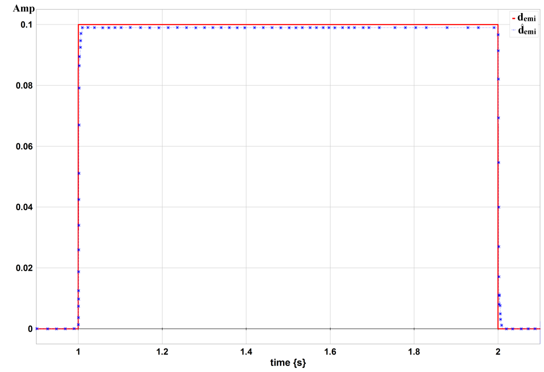

Figure 8: Perturbation signal d_{emi} and its estimation $\hat{d}_{emi}$.

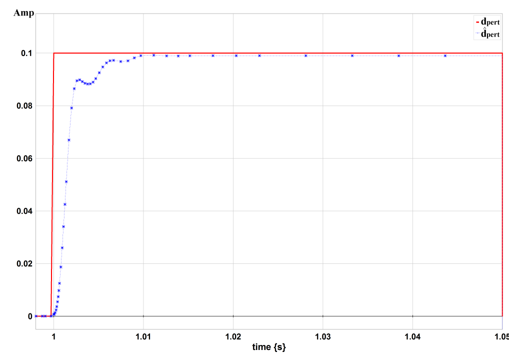

Figure 9: Zoom of perturbation signal d_{emi} and its estimation $\hat{d}_{emi}$.

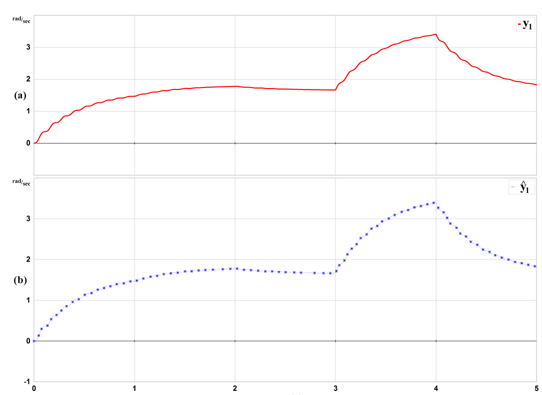

Figure 10: Output signal y_1 and the estimation $\hat{y}_1$.

or no well known parameters. In the literature, many solutions are based on the infinite structure of the model for the solvability conditions of the UIO and on the finite structure for the solutions with stability.

In this paper, a structural approach is proposed for the study of MIMO models which contains null invariant zeros. The innovative point is that estimation of the unknown input variables is also based on the integrals of measured

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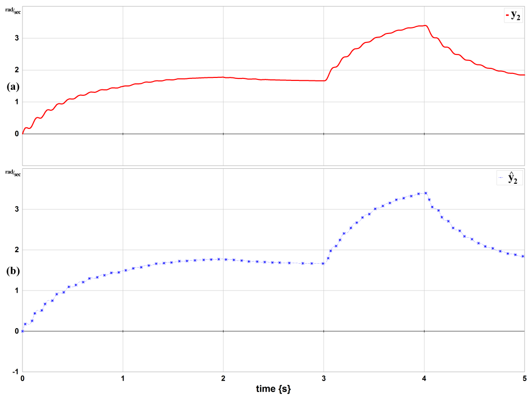

Figure 11: Output signal y_2 and the estimation $\hat{y}_2$.

variables, which allow to assign more modes in the state equation error estimation.

The simulations on an experimental bar system proved the effectiveness of the proposed UIO.

APPENDIX

A Structural Properties

A.1 Finite and infinite structures

Consider an invertible square model $\Sigma(C,A,B)$. The infinite structure of the multivariable linear model is characterized by different integer sets: $\{n_i'\}$ is the set of infinite zero orders of the global model $\Sigma(C,A,B)$ and $\{n_i\}$ is the set of row infinite zero orders of the row sub-systems $\Sigma(C_i,A,B)$. The row infinite zero order n_i verifies condition $n_i = \min \left\{k | C_i A^{(k-1)} B \neq 0\right\}$. n_i is equal to the number of derivations of the output variable $y_i(t)$ necessary for at least one of the input variables to appear explicitly. The global infinite zero orders are equal to the minimal number of derivations of each output variable necessary so that the input variables appear explicitly and independently in the equations. The infinite structure is also pointed out with the Smith-McMillan form at infinity of the transfer matrix. The finite structure of a linear model $\Sigma(C,A,B)$ is characterized by different polynomial matrices. Invariant zeros are pointed out with the Smith form of the System matrix associated to the state space representation.

A.2 Finite and infinite structures of bond graph models

Causality and causal paths are useful for the study of properties, such as controllability, observability and systems poles/zeros. Bond graph models with integral causality assignment (BGI) can be used to determine reachability conditions and the number of invariant zeros by studying the infinite structure. The rank of the controllability matrix is derived from bond graph models with derivative causality (BGD).

A LTI bond graph model is controllable iff the two following conditions are verified (Sueur and Dauphin-Tanguy 1991): first there is a causal path between each dynamical element and one of the input sources and secondly each dynamical element can have a derivative causality assignment in the bond graph model with a preferential derivative causality assignment (with a possible duality of input sources). The observability property can be studied in a similar way, but with output detectors. Systems invariant zeros are poles of inverse systems. Inverse systems can be constructed by bond graph models with bicausality (BGB) which are thus useful for the determination of invariant zeros.

The concept of causal path is used for the study of the infinite structure of the model. The causal path length between an input source and an output detector in the bond graph model is equal to the number of dynamical elements met in the path. Two paths are different if they have no dynamical element in common. The order of the infinite zero n_i for the row sub-system $\Sigma(C_i,A,B)$ is equal to the length of the shortest causal path between the i^{th} output detector z_i and the set of input sources. The global infinite structure is defined with the concepts of different causal paths. The orders of the infinite zeros of a global invertible linear bond graph model are calculated according to equation (25), where l_k is the smallest sum of the lengths of the k different input-output causal paths.

$$\left\{ \begin{array}{l} n_1' = l_1 \\ n_k' = l_k - l_{k-1} \end{array} \right. \tag{25}$$

The number of invariant zeros is determined by the infinite structure of the BGI model. The number of invariant zeros associated to a controllable, observable, invertible and square bond graph model is equal to $n - \sum n_i'$.

B Proof proposition 1

Consider the matrix product $N_{CL}F = A^{-1}F - A^{-1}F\Omega_d^{-1}\Omega_d - K[(C_1A^{n_1-1})^t(C_2A^{n_2-1})^t]^tF = -K\Omega$. Suppose that matrix Ω is not invertible. In this case, $\{n_1,n_2\} \neq \{n_1',n_2'\}$, i.e. the row infinite structure of system $\Sigma(C,A,F)$ is different of its global infinite structure. The rank of matrix $N_{CL}.F$ is equal to 1, thus matrix N_{CL} is not invertible and the observer cannot be synthesized.

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C Fixed Poles for the MIMO case, with a null invariant zero

Matrix N_{CL} in this MIMO problem is in equation (15). Pole placement is studied with the observability property of system $\Sigma([(C_1A^{n_1-1})^t,(C_2A^{n_2-1})^t]^t,N_{OL})$. Because the number of modes which can be assigned is equal to the rank of this observability matrix. The rows of the observability matrix of this system are calculated, firstly with the row matrix $C_1A^{n_1-1}$ associated with the null invariant zero, then with matrix matrix $C_2A^{n_2-1}$.

$$\begin{cases} C_1A^{n_1-1} \\ C_1A^{n_1-1}N_{OL} = C_1A^{n_1-1}\left(A^{-1}-A^{-1}F\cdot \right. \\ \qquad \left. \cdot \begin{bmatrix} C_1A^{-2}F \\ C_2A^{-1}F \end{bmatrix}^{-1} \begin{bmatrix} C_1A^{-2} \\ C_2A^{-1} \end{bmatrix}\right) = C_1A^{n_1-2} \\ C_1A^{n_1-1}(N_{OL})^2 = C_1A^{(n_1-3)} \\ \vdots \\ C_1A^{n_1-1}(N_{OL})^{n_1-2} = C_1A \\ C_1A^{n_1-1}(N_{OL})^{n_1-1} = C_1 \\ C_1A^{n_1-1}(N_{OL})^{n_1} = C_1A^{-1} \\ C_1A^{n_1-1}(N_{OL})^{n_1+1} = 0 \\ \vdots \\ C_1A^{n_1-1}(N_{OL})^{n-1} = 0 \end{cases} \tag{26}$$

A similar result is obtained for the row matrix $C_2A^{n_2-1}$, but $C_2A^{n_2-1}(N_{OL})^{n_2} = 0$.

Therefore, the non null rows of the observability matrix of system $\Sigma([(C_1A^{n_1-1})^t,(C_2A^{n_2-1})^t]^t,N_{OL})$ are thus: $[(C_1A^{-1})^t, C_1^t, (C_1A)^t, ..., (C_1A^{n_1-1})^t, C_2^t, (C_2A)^t, ...$ $, (C_2A^{n_2-1})^t]^t$. The rank of this matrix is equal to n_1+n_2+1 because model $\Sigma(C,A,F)$ is observable and for each output variable, the observability index is greater or equal to the row infinite zero order. The non null rows of the observability matrix of system $\Sigma([(C_1A^{n_1-1})^t,(C_2A^{n_2-1})^t]^t,N_{OL})$ are thus one part of the independent rows of the observability matrix of system $\Sigma(C,A)$. This rank can also be studied with the invariant subspaces defined in the geometric approach.

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