CROSS-DOCKING TRANSSHIPMENT PROBLEM APPROACHED BY NON LINEAR PROGRAMMING AND SIMULATION ANALYSIS

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ABSTRACT
The need for fast product delivery causes the attention of supply chain is addressed to strategies able to optimize the distribution process. In this field the cross-docking seems to be an efficient strategy which makes possible to reduce or eliminate the storage phase by meeting customer demand. In this paper a transshipment problem for cross-docking strategy is considered by means of a deterministic model studied through the non linear programming technique. The solution found allows to determine the optimal quantities to ship, the number of routes activated and the optimal truck number when the constraint on truck capacity is enforced. The influence of the demand fluctuation is also addressed through a simulation tool representing the cross-docking system. Finally the comparison between the cross-docking strategy and the direct delivery one is considered in terms of cost efficiency and trucks utilization.

Keywords: cross-docking, simulation, non linear programming model

1. INTRODUCTION

The increasing customer demand for fast product delivery leads business managers to improve the supply chain especially with reference to the distribution strategy. The minimization of the total cost of products delivered is related to the possibility of implementing an efficient control of the physical flow of products transferred. The specific importance of the distribution process is due to the fact that it can affect up to a 30% of an item price (Apte and Viswanathan, 2000). In order to implement new distribution strategies able to properly handle their products, industries nowadays look at the cross-docking as an efficient distribution strategy. Cross-docking can be defined as a continuous transportation where products are transshipped from the supplier, collected in the cross-dock, then are aggregated on the basis of their destination and finally shipped to the destination. The main objective of a cross-docking process is to avoid intermediate storage phases, thus eliminating inventory holding cost and labour intensive picking operations (Vahdani and Zandieh, 2010). Cross-docking attempts to lessen or even eliminate such burdens by reducing warehouses to purely trans-shipment centers where receiving and shipping are its only functions (Li et al., 2004). In other words, in the cross-docking network, the warehouses, as cross-docks, are transformed from inventory repositories to points of delivery, consolidation and pick-up (Chen et al., 2006). This allows to achieve a second objective consisting in the reduction of product cycle time (Li et al., 2009a). As observed by Yu and Egbelu (2008), the cross-docking systems operate best for companies which distribute a large amount of items and/or serve a large number of stores in a short time. With respect to the traditional warehouse systems the cross-docking allows to increase the inventory turnover, reduce the inventory level and operational costs and improve the customer responsiveness. As reported in Li et al. (2008), not all products are suitable for cross-docking and anyway the selection of the distribution strategy depends upon a number of factors such as product volume, product value, product life cycle, facility space constraint, stockout cost, etc. In particular the stockout costs are of greater importance in defining the products must be managed in the system due to the fact that generally in a cross-dock there is not inventory. For this reason usually cross-docking is suitable for fast moving items with stable demand, such as perishable products and agricultural products (Apte and Viswanathan, 2000). Groceries and agricultural products are also characterized by low stockout cost and for this reason they could be effectively managed with the cross-dock system. These products must be fast delivered to customer in order to preserve their freshness and because of the short period of circulation.
The implementation of a cross-docking system relates to the need to take decisions at different levels such as operational, tactical, and strategic levels. At the strategic point of view decisions address the determination of the optimal number of cross-dock and the number of trucks which must be disposed in the network considered. In such context Musa et al., (2010), proposed a Heuristic Algorithm to minimize the total transportation cost when each arch of the network can be satisfied through a direct link or one cross-dock

center. Charkhgard and Tabar (2011), faced the cross-docking problem in the case of determination of optimal truck capacity. They formulated a mixed integer non-linear programming model and solved it through a heuristic Algorithm such as the simulated annealing.
As reported in Agustina et al., (2010), the tactical level mainly relates to the determination of the best layout in the cross-docking. In this field Gue (1999), proposed a material flow model in order to minimize the flow inside the cross-dock, while Heragu (2005), proposed a model to simultaneously optimize the areas put in for storage, forward and cross-docking and the product allocation in order to minimize the total material handling cost.
The operational level is grouped in five research areas (Agustina et al., 2010) namely the scheduling problem, the transshipment problem, the dock door assignment problem, the vehicle routing problem and the product allocation problem. The scheduling problem usually aims at determining the optimal sequence of inbound and outbound trucks which in turn will minimize the makespan and then the total cost related to the products and trucks management. Larbi et al., (2007), studied the scheduling of transshipment operation in order to minimize the total inventory cost and truck replacement cost. He used a dynamic programming model and solved the model through a heuristic method. Li et al., (2009b), developed a truck scheduling with dock door assignment problem solved through a Genetic Algorithm. Boloori Arabani et al., (2011), proposed a multiobjective approach in order to minimize the makespan and the total lateness by means of a Genetic Algorithm. The transshipment problem concerns the determination of how much to ship, between which locations, on which routes and at what times. In this research field Lim et al., (2005), formulated a problem considering the inventory, the capacity of cross-docking and the time window constraints. Further studies have been conducted by Miao et al., (2008), who considered the transshipment problem where the transportations have fixed schedule and shipping and delivery can be only executed within time windows. The model's objective is minimizing the shipping and inventory holding cost by means of a Genetic Algorithm. The assignment problem deals with the proper assignment of inbound and outbound trucks within origins and destinations respectively. The first work in this field was realized by Tsui and Chang (1992). They developed a model to determine the assignment of receiving doors to the origins and shipping doors to the destinations. The objective of the model is to minimize the travel distance of the forklifts. Lim et al., (2006), considered an assignment problem with capacity of cross-dock and time window constraints. The objective of the model is to minimize the total shipping distance of transferring cargo from inbound to outbound dock. They solved the problem by means of a Genetic Algorithm.
In this paper the cost tradeoff between direct shipping and cross-docking systems is investigated referring to a numerical application solved by means of an Integer Non Linear Programming (INLP) approach. This approach is usually employed to simply represent complex systems and solve NP-hard problems as in the case of the cross-docking ones. In such context it is generally employed to determine a set of best candidate solutions that can be subsequently studied under disturbance conditions. In this study the optimal solution provided by the INLP approach is further tested by means of a post-optimality analysis performed through a simulation model in order to take into account the effects of the uncertainty of the demand on the Total Cost function and on the Utilization Coefficient of trucks. The robustness of the solution has therefore been evaluated.
Simulation can be defined as the process of designing a model of a real system, implementing the model as a computer program, and conducting experiments with the model for the purpose of understanding the behavior of the system, or evaluating strategies for the operation of the system (Smith, 1999). The simulation model takes the form of a set of assumptions concerning the operations of the system. These assumptions are expressed in mathematical, logical, and symbolic relationships between the entities, or objects of interest, of the system. Some of these assumptions can comprise those situations in which one or more inputs are random variables of the model and then they represent uncertainty elements that affect the system performances. The use of simulation allows to incorporate the randomness of such elements in the system, by representing the randomness through properly identified probability distributions arisen from the study of data related to the real processes of the system. In this case the outputs provided by the model can be considered only as estimates of the true characteristic of the model.
On the other hand there are some limitations affecting the use of simulation models and that must be taken into consideration when performing a simulation. First of all the real system could be very complex and several decisions must be taken in order to decide what details must be included in the model. Thus some details will be omitted and their effects lost or aggregated into other variables that are included in the model. In every case this representation will lead some inaccuracy sources. Another issue is the availability of data needed to describe the system behavior. In fact it is a common experience to describe a system by having few data. This issue must be considered prior to design the model in order to minimize its impact on the model itself.
The simulation-based approach played a significant role in analyzing performance at cross-docking centers. There are several studies where simulation modeled a cross-docking system. For example references to this application can be found in Rohrer (1995), that studied the importance of hardware and software system in the cross-docking systems. He describes how simulation helps to ensure success in cross-docking systems by determining optimal hardware configuration and

software control, as well as establishing failure strategies before cross-docking problems are encountered. Magableh and Rossetti (2005), studied a generic cross-docking facility with the aim at analyzing operational risks associated to individual cross-docking facility within a company's distribution network. Aickelin and Adewunmi (2008), proposed an assignment problem solved with the combined use of simulation and Memetic Algorithm. Liu and Takakuwa (2009), focused on the personnel planning of materials handling at a real cross-docking center in order to minimize the total personnel expenses at a cross-docking center. The approach employed includes the adoption of a simulation model together with integer programming. Arnaout et al., (2010), proposed a cross-docking simulation model in which the orders size and the due dates are represented by stochastic variables. Liu (2010), proposed a discrete event simulation model for non-automated cross-docking center with the aim of providing a decision making tool for logistic managers. Liu and Takakuwa (2010), studied the just-in-time shipments in a non-automated retail-cross-docking center. They proposed a simulation-based approach to analyze the material handling operation.

As you can see from the previous mentioned literature the simulation model has recently adopted to study the cross-docking problem under the view point of the variable affecting its performances. One of the variables that are usually poorly considered in cross-docking simulation is the variation of demand. The reason of the poor use of simulation tool in this field is due to the fact that usually the cross-docking problem is faced with reference to the deterministic behavior of the system modeled by considering that the customer demand is related to products having a stable demand and no fluctuations are considered. Furthermore it must be taken into account that the complexity of simulation models increases considerably as the number of suppliers, cross-docks and products increase as well. In fact as the number of nodes in the network increases the number of arches to be considered increase as number of suppliers* number of cross-docks + number of cross-docks*number of clients. In this paper a simulation tool has been employed to study the transshipment problem with cross-docking facilities where the hypothesis of deterministic behavior of the demand is relaxed and it is modeled as a stochastic variable. The comparison of the cross-docking strategy with the direct delivery one in terms of Average Total Cost and Trucks Utilization has been carried out by comparing the results of the two corresponding simulation models.

The remainder of the paper is hence organized as follows: Section 2 deals with the proposed methodology by presenting the two INLP models and the corresponding simulation models, thus the experimental application is showed in Section 3 and the main results are summarized. Finally the Section 4 reports the conclusions.

2. THE PROPOSED METHODOLOGY

The aim of the paper is to evaluate the performance of a traditional direct shipping transportation system and a cross-docking system, taking into account the effects of the uncertainty in the customers' demand by means of a simulation approach. However, solving an optimization problem by means of a simulation approach requires an excessive computational effort, therefore it is generally preferred to apply a two steps optimization procedure, where the best candidate solutions are determined first by means of a simplified model and a simulation approach is subsequently applied to select among the pre-determined best solution candidates.

The methodology here proposed, hence, consists in formulating a deterministic INLP model to determine the optimal solution of each problem neglecting the effects of uncertainty, and subsequently to perform a post-optimality simulation analysis.

2.1. INLP model for cross-docking transshipment problem

The INLP model for the cross-docking system has been formulated under the following notations:

i, number of suppliers
j, number of clients
n, number of cross-docks
k, number of products
C, maximum truck capacity
p_{jk}, demand of product *k* for the customer *j*
a_{ik}, availability of product *k* at the supplier *i*
d_{ij}, distance between the source *i* and the destination *j*
c_k, variable transport cost of the product *k* per unit distance from the origin (supplier or cross-dock) to the destination (cross-dock or client)
f_{ij}, fixed transport cost between the source *i* and the destination *j*. Such cost is proportional to the number of trucks routed between the source and the destination.
$\mathrm{M} = 100{,}000$, upper bound
$\mathrm{N} = 100{,}000$, upper bound

The following assumptions have been considered:

1. The suppliers and the cross-docks have very high capacity in order to ensure that products are always available and no stockout will occur.
2. Each supplier manufactures a single product.
3. The customer demand for each product is deterministic and constant and never exceeds truck capacity.
4. Trucks are always available and trucks have the same capacity.
5. Trucks have single destinations in a tour. They do not go from one destination node in the network to another but only from a origin (supplier or cross-dock) to a destination (cross-dock or client).
6. Trucks capacity and demand are expressed in terms of product units.

Decision variables:

q_{ikn}, transported quantity of product k from i to n
q_{knj}, transported quantity of product k from n to j
N_{in}, non negative variable representing the number of trucks on arc *i-n*
N_{nj}, non negative variable representing the number of trucks on arc *n-j*
x_{in} and x_{nj},binary variables

$$x_{in} = \begin{cases} 1, if\ the\ route\ i-n\ is\ active \\ 0 \quad otherwise \end{cases}$$

$$x_{nj} = \begin{cases} 1, if\ the\ route\ n-j\ is\ active \\ 0 \quad otherwise \end{cases}$$

The objective function:

$$\min z = \sum_{i,k,n} q_{ikn} * c_k * d_{in} + \sum_{k,n,j} q_{knj} * c_k * d_{nj} + \sum_{i,n} f_{in} * x_{in} * N_{in} + \sum_{n,j} f_{nj} * x_{nj} * N_{nj} \quad (1)$$

s.t.:

$$\sum_i a_{ik} \geq \sum_j p_{jk} \quad \forall\ k \quad (2)$$

$$\sum_n q_{ikn}\, x_{in} \leq a_{ik} \quad \forall\ i,k \quad (3)$$

$$\sum_{i,n} q_{ikn}\, x_{in} = \sum_j p_{jk} \quad \forall\ k \quad (4)$$

$$\sum_i q_{ikn} x_{in} = \sum_j q_{knj}\, x_{nj} \quad \forall n,k \quad (5)$$

$$\sum_n q_{knj} x_{nj} = p_{jk} \quad \forall j,k \quad (6)$$

$$\sum_{i,n,k} q_{ikn} = \sum_{k,n,j} q_{knj} \quad (7)$$

$$\sum_k q_{ikn} * x_{in} \leq C * N_{in} \quad \forall i,n \quad (8)$$

$$\sum_k q_{knj} * x_{nj} \leq C * N_{nj} \quad \forall n,j \quad (9)$$

$$x_{in} * M \geq \sum_k q_{ikn} \quad \forall i,n \quad (10)$$

$$x_{nj} * M \geq \sum_k q_{knj} \quad \forall j,n \quad (11)$$

$$x_{in} * N \geq N_{in} \quad \forall i,n \quad (12)$$

$$x_{nj} * N \geq N_{nj} \quad \forall j,n \quad (13)$$

All the variables must be non negative.

The objective function (1) is formulated to minimize the total transshipment cost consisting in both variable and fixed costs. The variable costs are proportional to the distance traveled and the quantity of products shipped, while the fixed costs are proportional to the number of trucks routed.
Constraint (2) ensures that the availability of product k at the suppliers is greater than the customer demand. Constraint (3) states that the quantity of product k sent by each supplier to the cross-docks is less than the availability of product k. Constraint (4) ensures that the total demand of the product k will be satisfied by the total quantity of product k shipped from i to n. Constraint (5) states that the quantity of product k which arrives at the cross-dock n is equal to the quantity of that product which leaves that cross-dock. Constraint (6) expresses the concept that the demand placed by each customer j must be entirely satisfied through the quantity which leaves the node n. Constraint (7) says that the total quantity picked from all the suppliers must be equal to the total quantity shipped to all clients. Constraints (8) and (9) ensure that the total quantity shipped respectively from a supplier or a cross-dock is equal to the capacity of a single truck multiplied for the number of trucks traveling the arc *i-n* or *n-j*. Constraints (10) and (11) ensure that the route *i-n* or *n-j* will be active only if at least a unit of product will be shipped. Finally Constraints (12) and (13) ensure that the number of truck is greater than zero only if the correspondent route is active.

2.2. INLP model for direct delivery transshipment problem

The direct delivery system consists in a network composed by i origins (suppliers) and j destinations (clients) in which each origin serves all the destination with a direct link. By assuming that each client will require a products mix and each origin makes only a product type there will be as many direct links from origin to destination how many clients will there be in the network. The model has been formulated under the following notations:

i, number of suppliers
j, number of clients
k, number of products
C, maximum truck capacity
p_{jk}, demand of product k for the customer j
d_{ij}, distance between i and j
c_k, variable transport cost of the product k per unit distance from the origin (supplier) to the destination (client)
f_{ij}, fixed transport cost between the source i and the destination j. Such cost is proportional to the number of trucks routed between the source and the destination.
M = 100,000, upper bound
N = 100,000, upper bound

The assumptions made for the cross-docking model result valid also for the present model by considering that the assumption 1is referred to the relation supplier-client.

Decision variables:

q_{ijk}, transported quantity of product k from i to j
N_{ij}, non negative variable representing the number of trucks on arc *i-j*
x_{ij}, binary variable

$$x_{ij} = \begin{cases} 1, if\ the\ route\ i-j\ is\ active \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

The objective function:

$$min\ z = \sum_{i,j,k} q_{ijk} * c_k * d_{ij} + \sum_{i,j} f_{ij} * x_{ij} * N_{ij} \qquad (14)$$

s.t.:

$$\sum_i a_{ik} \geq \sum_j p_{jk} \qquad \forall\ k \qquad (15)$$

$$\sum_j q_{ijk}\ x_{ij} \leq a_{ik} \qquad \forall\ i, k \qquad (16)$$

$$\sum_i q_{ijk}\ x_{ij} = \sum_j p_{jk} \qquad \forall\ j, k \qquad (17)$$

$$\sum_k q_{ijk} * x_{ij} \leq * C * N_{ij} \qquad \forall i, j \qquad (18)$$

$$x_{ij} * M \geq \sum_k q_{ijk} \qquad \forall i, j \qquad (19)$$

$$x_{ij} * N \geq N_{ij} \qquad \forall i, j \qquad (20)$$

All the variables must be non negative.

The objective function (14) is the same of the corresponding function in the cross-docking model. Similarly to the previous model constraint (15) ensures that the availability of product *k* at the suppliers is greater than the customer demand, while constraint (16) says that the quantity of product *k* sent by each supplier to the clients is less than the availability of product *k*.
Constraint (17) ensures that the total demand of the client *j* for the product *k* will be satisfied by the total quantity of that product shipped from *i* to *j*. Constraint (18) shows that the total quantity shipped from a supplier is equal to the capacity of a single truck multiplied for the number of trucks traveling the arc *i-j*. Constraint (19) ensures that the route *i-j* will be active only if at least a unit of product will be shipped. Finally constraint (20) ensures that that the number of truck is greater than zero only if the correspondent route is active.

2.3. Simulation Model-Building :Conceptual model definition and conceptual model translation for cross-docking and direct delivery systems

In order to built the simulation models the Model-Building step is performed consisting in the conceptual model definition and the conceptual model translation. The conceptual model definition can be expressed either formally (e.g. Activity Cycle Diagram) or informally (e.g. a list of assumptions) (Robinson, 1997). It aims at representing the actual operations carried out by the network (i.e. demand receiving, collection, shipping). The conceptual model definition for the cross-docking system is characterized by the assumptions 1-6 yet seen in the sub-sections 2.1. It must be pointed out that the assumption 1 implies that there will not occur queuing delays at the suppliers facilities neither stockout costs will be incurred. For the purpose of simulation the customer demand will be modeled as a random variable. Finally it must be added that the lead times between suppliers and cross-docks and cross-docks and clients will be considered null.
The conceptual models based on the previous discussed assumptions have been translated into simulation models through a C++ code. The assumptions have been accurately reproduced and the simulation models do not differ substantially from the conceptual models. Concerning the cross-docking system the conceptual model translation has been realized by considering the effort required to the simulation model. It arises from the deterministic model represented by the INLP model which is a NP hard problem whose complexity increases as the number of suppliers, cross-docks and clients increases as well. Consequently the effort required to determine the optimal solution of the INLP models and that required to the simulation models are very heavy. In order to reduce such effort the cross-docking network has been configured with only a cross-dock. This makes the INLP model simple to solve as well as the consequent simulation model. On the basis of this configuration of the network the simulation model has been realized.
The assumptions formulated in section 2.1 have been translated into the two simulation models in the following way:

1. The decision variables are constituted by the quantities transshipped, the number of routes activated and the number of trucks which travel along each route arisen from the optimal solution of the two NILP models.
2. The random demand which is an uncontrollable input variable of the model has been generated according to a normal distribution.
3. In the cross-docking simulation model each client in the system places an order of *k* different products. Thus the number of trucks needed is determined and the requested quantity will be sent from the suppliers to the cross-dock facility. Here the products are unloaded and collected on the basis of the customers products mix, the number of trucks needed to the shipment is determined and the products are sent to the clients.
4. For the direct delivery simulation model each client in the system places an order of *k* different products to each supplier. On the basis of the requested quantity the number of trucks needed will be calculated. Thus the requested quantity will be sent to the client.
5. The Total Cost and the Utilization Coefficient of trucks representing the output measures are determined.

2.4. Input data validation

Once the simulation model has been defined the next step to be realized consists in the validation of the input data consisting in the verification of the correspondence

between data collected with those achievable by a real system. In our case the data are represented by the number of nodes present in the network, the number of trucks routed, the distances between nodes, the customer demand and the variable and fixed costs.
The number of nodes of the network have been defined by starting from a real system and the distances between them have been arisen by starting from the Cartesian coordinates of each suppliers and clients. The localization of the cross-dock corresponds to the origin of the Cartesian plane. A qualitative representation of the cross-dock network is reported in Figure 1.

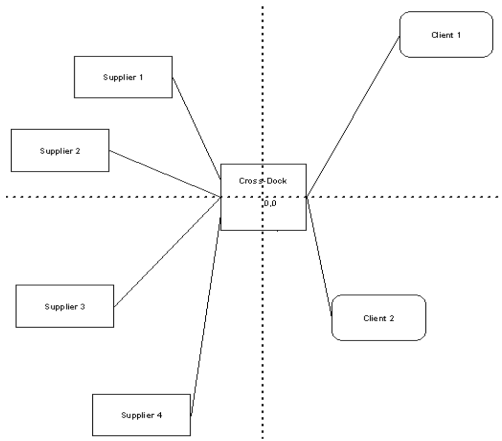

Figure 1. Cross-docking network

The active nodes as well as the number of trucks for each active route arise from the optimal solution of the INLP models. As regards the customer demand it has been modeled through a normal distribution to take into account the demand fluctuations for the products managed. This normal distribution is the result of the fitting of a time series analysis of food market demand.
Finally the variable and fixed costs have been defined by considering the data reported in a logistic review (TIR, 2010).

3. EXPERIMENTAL APPLICATION

In this section an experimental application of the discussed methodology is addressed consisting in running the two INLP models in order to get the optimal candidate solution of the two distribution systems considered and in the subsequent post-optimality analysis of the solutions carried out by means of the simulation tool. In both cases the models have been run by considering a general case of a network with four suppliers (S1-S4), two clients (C1-C2), one cross-dock (CD1) and four products (P1-P4). The input parameters consisting in the customers demand, the products availability and the distances are reported in Tables 1-5. The trucks capacity has been fixed equal to 70 units, the variable costs for products P1-P4 are respectively of 0.007€/unit*km, 0.005€/unit*km, 0.006€/unit*km, 0.003€/unit*km, while the fixed cost is equal to 250€ for the cross-docking strategy and 350€ for the direct delivery strategy.

Table 1. Customer demand for products P1-P4

Client/Product (unit/client)	P1	P2	P3	P4
C1	30	25	18	27
C2	40	25	30	25

Table 2. Products availability

Supplier/Product availability	P1	P2	P3	P4
S1	1,000	-	-	-
S2	-	1,000	-	-
S3	-	-	1,000	-
S4	-	-	-	1,000

Table 3. Distance Supplier-Cross-Docks

Distance Supplier/Cross-dock (km)	CD1
S1	100
S2	150
S3	170
S4	150

Table 4. Distance Cross-Docks-Clients

Distance Cross-Dock/Client (km)	C1	C2
CD1	150	110

Table 5. Distance Supplier-Client

Distance Supplier/Client (km)	C1	C2
S1	180	200
S2	220	250
S3	210	225
S4	190	160

At first the two INLP models have been solved by using LINGO software. Results show that a feasible solution can be found in both cases and the routes, the quantity to ship, the number of trucks that must travel along each route and the Total Cost have been determined. Results are reported in Table 6, 7 and 8.

Table 6. Results of the INLP model for Cross-Docking

Cross-Docking-INLP Model Solution	
Q_IKN(S1, P1, CD1)	70
Q_IKN(S2, P2, CD1)	50
Q_IKN(S3, P3, CD1)	48
Q_IKN(S4, P4, CD1)	52
X_IN(S1-CD1)	1
X_IN(S2-CD1)	1
X_IN(S3-CD1)	1
X_IN(S4-CD1)	1
N_IN(S1-CD1)	1
N_IN(S2-CD1)	1
N_IN(S3-CD1)	1
N_IN(S4-CD1)	1
Q_KNJ(P1, CD1, C1)	30
Q_KNJ(P1, CD1, C2)	40
Q_KNJ(P2, CD1, C1)	25
Q_KNJ(P2, CD1, C2)	25
Q_KNJ(P3, CD1, C1)	18

Q_KNJ(P3, CD1, C2)	30
Q_KNJ(P4, CD1, C1)	27
Q_KNJ(P4, CD1, C2)	25
X_NJ(CD1-C1)	1
X_NJ(CD1-C2)	1
N_NJ(CD1-C1)	2
N_NJ(CD1-C2)	2

Table 7. Results of the INLP model for Direct Delivery

Direct Delivery-INLP Model Solution	
Q_IJK(S1, C1, P1)	30
Q_IJK(S1, C2, P1)	40
Q_IJK(S2, C1, P2)	25
Q_IJK(S2, C2, P2)	25
Q_IJK(S3, C1, P3)	18
Q_IJK(S3, C2, P3)	30
Q_IJK(S4, C1, P4)	27
Q_IJK(S4, C2, P4)	25
X_IJ(S1-C1, S1-C2)	1
X_IJ(S2-C1, S2-C2)	1
X_IJ(S3-C1, S3-C2)	1
X_IJ(S4-C1, S4-C2)	1
N_IJ(S1-C1, S1-C2)	1
N_IJ(S2-C1, S2-C2)	1
N_IJ(S3-C1, S3-C2)	1
N_IJ(S4-C1, S4-C2)	1

Table 8. Output of interest measures for Cross-Docking and Direct Delivery

	Cross-Docking		Direct Delivery
Total Cost (€)	2,310.06		3,043.12
Total Variable Cost (€)	310.06		243.12
Total Fixed Cost (€)	2,000		2,800
	Route S-CD	Route CD-C	Route S-C
Number of routes active	4	2	8
Number of trucks	4	4	8
	S-CD	CD-C	S-C
Mean Utilization Coefficient	0.7857	0.7857	0.3928

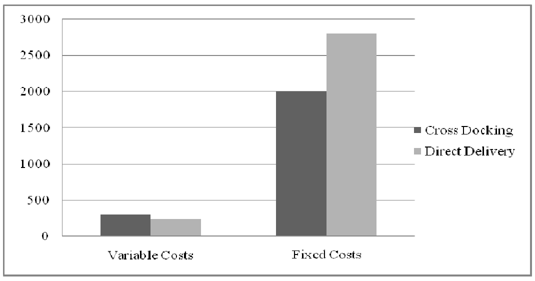

Figure 2. Comparison of Variable an Fixed Costs between the two strategies

As shown in Table 8 the cross-docking outperforms the direct delivery. In particular by analyzing the Total Cost (see also Figure 2) it can be observed that the cross-docking has lower fixed costs compared to the direct delivery, while the variable costs are greater that the direct delivery. This can be explained by considering that the fixed costs depend on the number of trucks routed and on the routes activated that in the case of cross-docking are less than the direct delivery case. The variable costs are lower in the direct delivery strategy because they depend on the total distances traveled which are lower in the this case. Finally the Utilization Coefficient is greater in the cross-docking strategy than the direct delivery one. This can be explained by considering that in the cross-docking strategy the number of routes activated between the suppliers and the cross-dock and the cross-dock and the client are lower than the total number of routes activated in the direct delivery strategy between suppliers and clients. This involves a less number of trucks will be routed in the cross-docking strategy in each supplier-cross-dock and cross-dock-client route as compared to the total number of trucks managed in the direct delivery strategy.

Successively the best candidate solutions found for the two INLP models have been employed to perform a post-optimality analysis consisting in running the two simulation models yet discussed in section 2.3. For the purpose of the present study it aims at showing the sensitivity of solutions found in the case in which the customer demand is subject to fluctuations. The only uncontrollable input variable of the simulation models is the normal distribution representing the customer demand whose mean is equal to the customer demand of the deterministic case (Table 1) while the standard deviation is reported in Table 9.

Table 9. Standard deviation of demand

Client/Standard Deviation of Product	P1	P2	P3	P4
C1	8	9	5	6
C2	7	5	4	9

For the cross-docking model once the customer’s order for each product is placed each supplier will send to the cross-dock a supply equal to the total customer demand for the single product considered. Each of the routes which join the suppliers with the cross-dock results active. The number of trucks traveling along each route is determined by dividing the quantity shipped by each supplier for the truck capacity. Once the products arrive at the cross-dock they are consolidated on the basis of the customer requests. Thus the number of trucks leaving the cross-dock is determined by dividing the total product demand of each customer for the truck capacity.

For the direct delivery model the only difference is that at the arriving of the customer demand each supplier will determine the trucks number by dividing each customer demand for the truck capacity and it will send

directly to the customer the required quantity. Each route joining the suppliers with the customers results active.
Ten thousand replications of the two simulation models have been carried out to ensure a 99.5% of accuracy of the interest measures which are the Average Total Cost and the Average Utilization Coefficient of the trucks routed. The number of runs needed to ensure the accuracy has been determined by running at first ten replications of the two simulation models and by calculating the number of runs needed to ensure the desired precision by means of the following formula:

$$n_{\alpha}(\varepsilon, \alpha) = \frac{4\sigma^2 z_{\frac{\alpha}{2}}^2}{\varepsilon^2} \quad (21)$$

where:
σ is the standard deviation of the interest measures on the basis of the initial ten run,
$z_{\alpha/2}$ is the normal random variable of a standard normal distribution corresponding to the precision required 1-α,
ε is the absolute width of the confidence interval referred to the ten run and determined as:

$$\varepsilon_{\alpha}(\alpha) = \frac{2 z_{\frac{\alpha}{2}} \sigma}{\sqrt{n}} \quad (22)$$

where *n* is the initial number of runs.
For detailed discussion about statistic aspect refer to Whitt (2005). The results are showed in Tables 10 and 11.

Table 10. Results of simulation for Cross-Docking

Cross-Docking Model			
	Mean	Standard Deviation	Precision
Average Total Cost (€)	2,457.59	189.762	0.996
Average Total Variable Cost (€)	310.14		
Average Total Fixed Cost (€)	2,147.45		
Route S-CD	4		
Route CD-C	2		
	S-CD	CD-C	
Mean Number of Trucks	(4.54)	(4.04)	
Mean Utilization Coefficient	0.697	0.77880	0.996

Table 11. Results of simulation for Direct Delivery

Direct Delivery Model			
	Mean	Standard Deviation	Precision
Average Total Cost (€)	3,037.56	37.6239	0.999
Average Total Variable Cost (€)	239.94		
Average Total Fixed Cost (€)	2,797.62		
Route S-C	8		
	S-C		
Mean Number of Trucks	8		
Mean Utilization Coefficient	0.3931		

The stochastic scenario shows that the Average Total Cost of the cross-docking strategy is less than the direct delivery one similarly to the case of deterministic configuration. In the stochastic case the average percentage of saving cost is about of 19%. It is worth to underline that the standard deviation of the Average Total Cost is greater in the cross-docking strategy as compared to the direct delivery one as you can see also from Figures 3 and 4. Such figures report the frequency of the Total Cost for the 10,000 replications of the two simulation models. They underline that the Total Cost function in the cross-docking strategy ranges between 1,950€ and 3,750€, while in the direct delivery case ranges from 2,600€ and 3,500€. The greater standard deviation of the cross-docking system can be explained by considering that the Utilization Coefficient value is greater in the cross-docking strategy compared to the direct delivery one. In fact when the customer demand increases due to the demand fluctuation the number of trucks in the cross-docking system tends to increase as well by causing the increasing of the Average Total Cost, while in the direct delivery strategy the number of trucks tends to be always the same due the low Utilization Coefficient and the Average Total Cost increasing is very low.

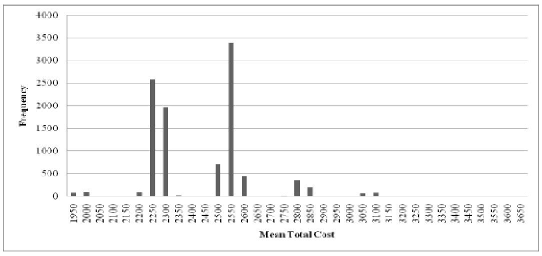

Figure 3. Frequency of Total Cost for Cross-Docking

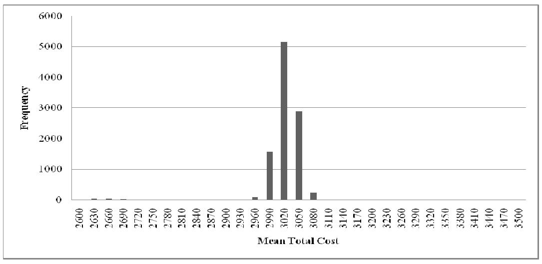

Figure 4. Frequency of Total Cost for Direct Delivery

At the end of the experimental analysis a sensitivity analysis has been conducted by varying the Average

Customer Demand of ±5% and of ±10% to show the impact of demand fluctuation on the Average Total Cost, the Average Standard Deviation and the Average Utilization Coefficient. Results are reported in Tables 12 and 13.

Table 12. Sensitivity analysis for Cross-Docking

Sensitivity analysis-Cross-Docking Model (-10%, -5%,+5%, +10%)		
	Mean	Standard Deviation
Average Total Cost (€)	(2,324.7, 2,380.94, 2,539.5, 2,634.7)	(162.64, 170.5, 213.9, 238.50)
	S-CD	CD-C
Average Number of Trucks	(4.26, 4.37, 4.72, 4.90)	(3.92, 3.97, 4.13, 4.27)
Average Utilization Coefficient	(0.667, 0.686, 0.7050, 0.711)	(0.723, 0.75, 0.80, 0.814)

Table 13. Sensitivity analysis for Direct Delivery

Sensitivity analysis-Direct Delivery Model (-10%, -5%,+5%, +10%)		
	Mean	Standard Deviation
Average Total Cost (€)	(3,010.4, 3,024.0, 3050.5, 3048.2)	(50.41, 44.2, 31.8, 29.6)
	S-C	
Average Number of Trucks	(7.98, 7.98, 8, 7.99)	
Average Utilization Coefficient	(0.354, 0.373, 0.412, 0.43)	

Results show that in both the case of cross-docking and direct delivery strategies an increasing in the customer demand causes the Average Total Cost, the Average Number of trucks and the Average Utilization Coefficient increase as well. On the other hand the Average Standard Deviation increases with the customer demand in the case of cross-docking strategy while decreases in the case of direct delivery one. This means that the cross-docking strategy is more sensitive to the fluctuation of the demand than the direct delivery. The percentage of variation of the Average Total Cost, the Average Number of Trucks and of the Average Utilization Coefficient for a variation of ±5% and ±10% of the Average Customer Demand for the two strategies is reported in Table 14. The variation is calculated to respect to the average values already seen in Tables 10 and 11.

Table14. Percent variation of the interest measures

		Absolute variation Cross-Docking		Absolute variation Direct Delivery
Average Total Cost	-10%	5.71%		0.9022%
	-5%	3.21%		0.4484%
	5%	3.22%		0.4242%
	10%	6.72%		0.3491%
	Average variation	4.72%		0.5310%
		S-CD	CD-C	S-C
Average Number of Trucks	-10%	6.57%	3.06%	0.251%
	-5%	3.89%	1.76%	0.251%
	5%	3.81%	2.17%	0.000%
	10%	7.34%	5.38%	0.125%
	Average variation	5.40%	3.09%	0.157%
Average Utilization Coefficient	-10%	4.50%	7.72%	11.05%
	-5%	1.60%	3.84%	5.39%
	5%	1.13%	2.65%	4.59%
	10%	1.97%	4.32%	8.58%
	Average variation	2.30%	4.63%	7.40%

Results show that the average variation of the Average Total Cost is equal about to 4.72% in the case of cross-docking and only to 0.53% in the case of direct delivery strategy. However the Average Total cost of the cross-docking strategy results always less than that of direct delivery. The average variation of Average Number of Trucks is equal about to 5.40% and 3.09% in the case of cross-docking and only to 0.15% in the case of direct delivery. This confirm the greater sensitivity of the cross-docking strategy to respect these two measures. In fact when the average demand increases/decreases the Average Number of Trucks increases/decreases as well and the Average Total Cost consequently (Tables 12 and 13). On the contrary the Average Utilization Coefficient varies about of 2.30% and 4.63% in the case of cross-docking and of 7.40% in the case of direct delivery. This substantially confirms that in the case of cross-docking strategy the number of trucks tends to increases as the average demand increases as well, while in the case of direct delivery when the Average Utilization is low, the number of trucks tends to be always the same and consequently the Average Utilization increases (Tables 12 and 13).

4. CONCLUSIONS

In this paper a transshipment problem for the cross-docking system has been addressed. At first a deterministic solution of the problem has been found by means of a INLP model and subsequently by starting from this optimal solution a post-optimality analysis has been conducted by means of the simulation approach in order to take into account possible fluctuations in the customer demand. The cross-docking system has been also compared to the direct delivery strategy. Results show that the cross-docking strategy outperforms the direct delivery one as regard the optimization of the Total Cost in both the deterministic and the stochastic case.

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