OPTIMAL CROP SELECTION AND SCHEDULING USING A GENERIC CROP GROWTH MODEL AND INTEGER PROGRAMMING MODEL

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ABSTRACT
Crop growth programs and crop optimizers act as important decision support tools in selecting crops and influencing farmer decision process. However the crop optimizers do not always include the crop growth models while selecting the best possible crops. In this paper we will present the equations used to model a generic crop growth model and couple it with an optimal crop scheduler, to select the best possible sequence of crops. The scheduler aims to achieve various objectives like maximizing performance, maximizing economic gain and minimizing environmental impact. Each objective has three different configurations with regards to the crop rotation and crop growth period. The results will be visualized using a Gantt chart to show the scheduling of the crops.

Keywords: crop growth program, optimal crop scheduling, decision support system, integer programming

1. INTRODUCTION

The definition of sustainability in agriculture has been subject to multiple revisions. Various interpretations and objectives have been prescribed to define sustainability. Since, the conditions dictating agricultural activities vary between different regions, varying evaluation criteria are required to judge sustainability. This definition has progressed from a purely economic objective to now include ecological and social considerations. Sustainable agriculture is now considered as, methods or practices that facilitate the development of social, economic and environmental objectives by finding a common ground between the various conflicting options that these objectives present.

It is now necessary for policy makers and farmers to understand the decisions being made at farm level, and their consequences on the immediate environment, in order to create long-term plans for sustainable agriculture.

This need for modeling various scenarios and decisions at farm level, and analyzing their impact, has resulted in the demand for expert systems that can aid farmers and decision makers in making decisions that meet the objective of sustainability.

Such a system would need to combine the various aspects of farm level procedures from crop growth dynamics to community based decision models. It would require quantifying various decision alternatives and scenarios through data analysis and a review of previous work. The designers of such a system would also need to identify those areas of farm processes that have a significant impact on the farm level decision process, and eliminate excessive complexity in the system. In order to give decision makers access to all these various aspects of decision making, we first need to understand the various socioeconomic and environmental issues faced by farmers, and derive the criteria to measure sustainability.

Typically farmers are profit maximizers. Their primary objective is to maximize their profits for each cropping season. Social and environmental welfare are generally treated as secondary objectives that are contingent upon the completion of the primary objective. The farmers often face various issues in achieving their primary objective. Additionally, the actions taken during the pursuit of the primary objective can cause a significant impact on the secondary objectives.

A tool, which is aimed at helping farmers make decisions, needs to be able to present the problem to the user from multiple perspectives and provide solutions for each of those perspectives. It should also be able to help the decision makers compare the results of the different perspectives, and provide a measure for computing the best decision, or sequence of decisions.

Decision support systems for farmers, fall under this category of expert systems. Due to the multiple methods of formulation of a farming problem, no single modeling methodology can answer all the questions a decision maker might ask. The various decision modeling methods can only address specific sets of scenarios. For example, Bazzani et al., 2005, Berentsen, 2003, El-Nazer & McCarl, 1986 treat farming problems as resource and policy optimization problems. They do not address the motivations behind decision making processes explicitly.

On the other hand, Bosma, Kaymak, van den Berg, Udo, & Verreth, 2011, Fairweather, 1999, Rehman & Romero, 1993 address farming problems as purely theoretical decision making problems. This causes the models to over stress the importance of some variables, which might not have an actual impact in real world scenarios.

Additional problems arise when most models do not integrate crop growth models into their decision support systems. This problem stems from research groups, which concentrate on specific problems of specific areas. Though this gives the research groups the flexibility to use historic yield data while formulating their problem, it becomes hard to apply their conclusions to other regions and crops.

Our research effort stems from this need to design a decision support system that integrates multiple modeling methodologies into a single system. Such a system should not require unrealistic amount of inputs from users, who have a limited knowledge of the various methodologies. Fig. 1 is a conceptual model of the architecture of the proposed system.

The crop growth model is the base component of the architecture. It consists of the necessary equations to model the growth of crops. It is capable of simulating the yield and also the height of a set of crops. The output of this layer is used in both the optimizer and the individual decision model. The optimizer is the second model of the architecture. Its purpose is to compute the optimum crop/combination of crops that the farmer can plant in order to maximize his profits, while maintaining a certain level of environmental friendliness.

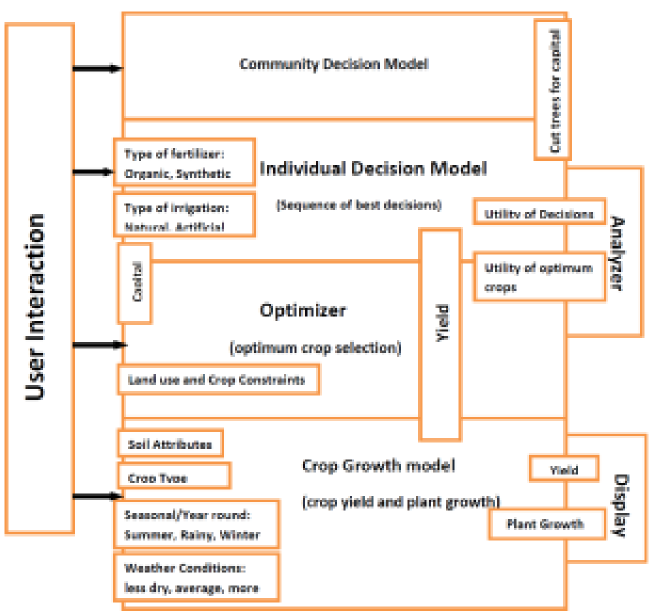

Figure 1: Multi layered decision support system for farmers

The individual and community decision models are used to simulate individual and collective farmer decision making process. These layers can show the impact of farmer decisions on their economic standing and the environment.

A generic crop growth model was analyzed by the authors, Vegesana & McKenzie, 2013, to determine the important variables affecting crop growth. A mathematical programming model was also implemented by Collins et al., 2013 to determine optimal crop selection and rotations for a small set of crops. An agent based decision models was explored by Vegesana & McKenzie, 2014 for use in the individual and community layers.

In this paper we will couple the crop growth model with the optimizer layer to evaluate the optimum scheduling of crops for a given set of weather data. The simulation results will be presented using a Gantt chart for various configurations and objectives. In the next section we will present the various equations used to model the crop growth and the applications of this model. In section 3 the mathematical programming model for optimum selection and scheduling of the crops will be presented. Section 4 contains the results for the simulation, and we conclude with some observations for future work in section 5.

2. CROP GROWTH MODEL

The mathematical model necessary for crop growth has been developed from existing resources. Several mathematical models are available to simulate the growth pattern of various crops McCown, (2002), Teh, (2006). The drawback of these models is that they are crop specific. Since our project did not need the complexity of the various crop specific models, we have attempted to use a generic crop growth model to simulate the crop bio mass yields and plant height. In this section, we will look at a form of the crop biomass equation. The individual variables in the equation will be explored, to see how we have arrived at the final form of the equation.

2.1. Generic Crop Growth Equation

A generic equation for plant biomass growth(Teh, 2006) can be written as:

$$\frac{dW}{dt_i} = \varepsilon * 0.5\, Q_0 \,[1 - e^{(-k * LAI_{i-1})}] * 0.0001 \qquad (1)$$

$\frac{dW}{dt_i}$ is the daily increment in biomass weight for the crops, in tonnes/hectare (ta/ha). Q_0 is the daily solar radiation in MJ m^{-2} d^{-1}. ε is the radiation use efficiency that converts the daily radiation into photosynthetically active radiation that is used by the plants. This co-efficient is crop specific. k is the

extinction co-efficient. It is generally assumed to have a value of 0.65 for all crops.
LAI_{i-1} is the leaf area index for the previous day. LAI is a dimensional quantity that represents the one sided green leaf area per unit ground surface. In order to evaluate LAI, we need to calculate heat units, heat unit index, and heat unit factor for each day. A crop starts growing once the daily average temperature exceeds the base temperature for the crop. Daily heat unit is the difference between the daily average temperature and the base temperature required for germination. Heat unit HU is given by Williams, Izaurralde, & Steglich, 2008 as:

$$HU = T_{Avg} - T_{base} \quad (2)$$

Each day, if the value of the heat unit is greater than zero, it is accumulated as part of the total heat units HU_{tot}. These accumulated heat units are divided by the potential heat units for a crop to arrive at the heat unit index HUI

$$HU_{tot} = HU_{tot} + HU \quad \text{for } HU > 0 \quad (3)$$

$$HUI = \frac{HU_{tot}}{HU_{pot}} \quad (4)$$

The potential heat units for a crop are calculated by multiplying the difference between the optimal and base temperatures for a crop with the total number of growing days.

$$HU_{pot} = Planting\ duration * (T_{opt} - T_{base}) \quad (5)$$

HUI is a value between 0 and 1 that is used to measure the progress of a crops growth as a function of the daily temperature. It is also used to calculate the heat unit factor HUF, which indicates the fraction of the maximum leaf area index for the current heat unit index.

$$HUF_i = \frac{HUI}{HUI + \exp(ah_1 - ah_2 * HUI)} \quad (6)$$

$$ah_2 = \frac{\ln\left(\frac{frp_1}{frl_1} - frp_1\right) - \ln\left(\frac{frp_2}{frl_2} - frp_2\right)}{frp_2 - frp_1} \quad (7)$$

$$ah_1 = \ln\left(\frac{frp_1}{frl_1} - frp_1\right) + ah_2 * frp_1 \quad (8)$$

frp_1, frl_1, frp_2, frl_2 are crop specific parameters that provide the fraction of the maximum leaf area index reached for a specific period in the growing stages. These values are regression co-efficients that researchers have determined experimentally to fit the leaf development curve.
Finally, the leaf area index for each day is given by Neitsch, Arnold, Kiniry, & Williams., 2005 as:

$$LAI_i = \begin{cases} LAI_{i-1} + dH_{F,i} * LAI_{max} * (1 - \exp(5 * (LAI_{i-1} - LAI_{max}))), & i < decline\ period \\ LAI_{i-1} * \frac{1 - HUI}{1 - HUI_{sen}}, & i \geq decline\ period \end{cases} \quad (9)$$

$$dH_{F,i} = HUF_i - HUF_{i-1} \quad (10)$$

HUI_{sen} is the heat unit index when the crop enters its decline stage. During the growth stages the LAI is an exponential function of the LAI from the previous day and the maximum leaf area index LAI_{max}. Once the crop starts declining, the leaf area also starts declining as a function of the heat unit index.

2.2. Evapotranspiration

Evapotranspiration is the combined process of plant transpiration and soil evaporation. Plants lose almost 99% of the water they take up due to evaporation. This process is called transpiration. Simultaneously, the soil surface also undergoes evaporation and loses water to the atmosphere.
Evapotranspiration is used as a means to calculate the water requirement of a crop for each day during its life cycle. Evapotranspiration is heavily influenced by the climate conditions. It is high in hot and dry conditions, and low in cloudy and cool areas. Crop evapotranspiration for each day is calculated by first calculating the potential evapotranspiration. Potential evapotranspiration is defined as the evapotranspiration that would occur from a large area uniformly covered with green vegetation with an unconstrained access to water.
Various methods have been developed to calculate the potential evapotranspiration on any given day. The Penman model, the Penman-Monteith model, the Priestly-Taylor model, and the Hargreaves model have all been successfully used to calculate daily evapotranspiration. The current crop growth model implements the Penman model to calculate the evapotranspiration. The Penman model calculates the evapotranspiration for a short green crop, like grass, that uniformly covers the surface of the land and has unconstrained water supply.

Table 1: Crop Coefficients and growth stages

Crop	kcini	kcmid	kclate	Initial duration	Development duration	Mid-stage duration	Decline duration
Broccoli	0.15	0.95	0.85	135	35	45	40
Lettuce	0.15	0.9	0.9	140	25	30	65
Onions	0.15	0.95	0.65	150	30	40	60

The equation for the penman model is given by Williams et al., 2008 as:

$$E_O = \frac{\Delta * R_N + psychro * FWV * VPD}{H_V * (\Delta + psychro)} \quad (11)$$

E_O is the potential evapotranspiration for any given day, measured in mm/day. Δ is the slope of the saturation vapor pressure curve in kPa/^oC. Vapor pressure is the amount of pressure exerted by vapor in a closed container. It is an indication of the rate of evaporation of water from the soil surface. The slope of the vapor pressure curve indicates the speed with which the surface water content of the soil is evaporating. It is an exponential function of the daily average temperature in ^oC, given by the formula:

$$\Delta = 25029.4 * \frac{exp\left(\frac{T_{Avg} * 17.269}{T_{Avg} + 237.3}\right)}{(T_{Avg} + 237.3)^2} \quad (12)$$

FWV is a function of the wind speed W_S in m/s, that calculates the aerodynamic conductance of air in mm/kPa*day. It is calculated using the formula:

$$FWV = 2.7 + 1.63 * W_S \quad (13)$$

VPD is the vapor pressure deficit in kPa. It is used to measure the difference in the actual water vapor pressure E_a, and the vapor pressure at saturation E_s, for the daily average temperature T_{Avg} and relative humidity R_h expressed as a fraction.

$$VPD = E_s - E_a \quad (14)$$

$$E_s = 6.1078 * exp\left(\frac{T_{Avg} * 17.269}{T_{Avg} + 237.3}\right) \quad (15)$$

$$E_a = E_s * R_h \quad (16)$$

$psychro$ is a psychrometric constant. It is useful in relating pressure P_B, in kPa/^oC, of water in air to a specific temperature. It is given by the formula:

$$psychro = P_B * 7.2063 * 10^{-4} \quad (17)$$

H_V is the latent heat of vaporization of water at the daily average temperature T_{Avg}.

$$H_V = 2.501 - 0.0022 * T_{Avg} \quad (18)$$

The potential evapotranspiration calculated in the previous step is for a reference crop like grass or alfalfa. To scale this value to a specific crop, and to calculate its daily water use, we need to multiply the potential evapotranspiration, E_O, value with the crop co-efficient K_C.

$$E_S = E_O * K_C \quad (19)$$

The crop co-efficient K_C depends upon the crop type, the growth stages of the crop, and the climate. The general crop co-efficient encompasses the evaporation from both the crop and the soil. General values of the co-efficient are available, and can be used to calculate the daily water requirement. If we need to calculate the daily crop co-efficient by taking the soil type into account, we will need to split the co-efficient into the crop specific co-efficient, and the soil co-efficient. The crop co-efficient K_C is given by Allen, Pereira, Raes, & Smith, 1998 :

$$K_C = K_s * K_{cb} + K_e \quad (20)$$

K_{cb} is the basal crop co-efficient. For every crop, this value is defined for the different crop growth stages: initial, development, middle, and decline. It is important to know the duration of each of these stages for each crop, and the respective co-efficient. Table 1 shows a sample of the basal crop co-efficient for different crops, at the different growth stages.

The soil co-efficient K_e is calculated using the formula:

$$K_e = K_r * (1.21 - K_{cb}) \quad (21)$$

The values K_r, K_s are evaporation reduction co-efficients that are dependent on the depth of the water depleted from the top soil for the crops. These co-efficients are given by the following formulae:

$$K_r = \begin{cases} \frac{TEW - D_{e,i-1}}{TEW - REW}, & D_{e,i-1} > REW \\ 1, & D_{e,i-1} < REW \end{cases} \quad (22)$$

$$K_s = \begin{cases} \frac{TAW - D_{e,i-1}}{TAW - RAW}, & D_{e,i-1} > RAW \\ 1, & D_{e,i-1} < RAW \end{cases} \quad (23)$$

TEW and REW are the total and readily evaporable water levels respectively, in mm, for different soils. TAW and RAW are total and readily available water levels each day, in mm, for a given crop-soil combination. TEW, and REW values are readily available for major soil types. TAW, and RAW are given by:

$$TAW = 1000(\theta_{FC} - 0.5 * \theta_{WP}) * \ Z_r \quad (24)$$

$$RAW = p * TAW \quad (25)$$

θ_{FC}, and θ_{WP} are the water content of each soil at field capacity and wilting point respectively. These values are constants for each soil. Z_r is the root depth of the crop at each day. p is a crop specific constant that is used to calculate RAW from TAW. The following table lists θ_{FC} and θ_{WP} values for all the major soil types.

Table 2: Coefficients for various soils

Soil type	θ_{FC}	θ_{WP}	**REW**	**TEW**
Sand	0.12	0.04	5	10
Loamy sand	0.16	0.06	6	13
Sandy loam	0.24	0.11	8	18.5
Loam	0.26	0.12	9	20
Silt loam	0.3	0.14	10	23
Silt	0.33	0.17	10	24.5
Silt clayloam	0.32	0.2	10	22
Silty clay	0.37	0.23	11	25.5
Clay	0.37	0.22	11	26

2.3. Nutrient Requirements

Crops require nitrogen and phosphorous for proper growth. The model calculates the potential nitrogen and phosphorous content of the crop for each day. The nutrient demand is then calculated by subtracting the actual content from the potential content. This nutrient demand is the amount of fertilizer required for a stress free growth. The potential content for each day is given by the formula:

$$N_{pot} = \ W_i * (bn_1 + \ bn_2 * \exp(-bn_3 * HUI) \quad (26)$$

$$P_{pot} = \ W_i * (bp_1 + \ bp_2 * \exp(-bp_3 * HUI) \quad (27)$$

N_{pot}, and P_{pot} are the potential content for a given day. bn_1, bn_2, bn_3, bp_1, bp_2 and bp_3 are crop specific parameters that express the optimal N and P concentrations as a function of the heat unit index.

2.4. Stress factors

Under ideal conditions the crop growth is stress free and the crop is able to achieve its maximum possible growth for each day. However, actual crop growth suffers from multiple forms of stress. Lack of sufficient water, sub-optimal temperature, and a lack of nutrients inhibit daily crop growth. This is modeled in the equations by multiplying the daily biomass with a stress factor. The new daily biomass is given by

$$\frac{dW}{dt_i} = \text{Stress}* \frac{dW}{dt_i} \quad (28)$$

Stress is a value between 0 and 1 that scales down the daily biomass to actual values. There are various kinds of stress acting on the crop. These are water stress, temperature stress, nitrogen stress and phosphorous stress. Stress is given by

$$Stress = \min(water, temperature, nitrogen, phosphorous\ stress) \quad (29)$$

Water stress is the ratio of the available water content to the actual water necessary. It is given by:

$$Water\ Stress = \frac{RAW}{Required\ water\ content} \quad (30)$$

Temperature stress is a sinusoidal function of the daily average temperature, optimal temperature and the base temperature of the crop.

$$Temp\ Stress = \sin(1.5707 * \frac{(T_{Avg} - T_{base})}{(T_{Opt} - T_{base})}) \quad (31)$$

Nutrient stress for both phosphorous and nitrogen is expressed as a function of the ratio of the actual nutrient content to the optimal nutrient content.

$$Nitrogen\ stresss = 200 * \frac{\frac{Actual\ nitrogen\ content}{Optimal\ Nitrogen\ content}}{(\frac{Actual\ nitrogen\ content}{Optimal\ Nitrogen\ content} + \exp\left(4.065 - 0.0535 * \frac{Actual\ nitrogen\ content}{Optimal\ Nitrogen\ content}\right))} \quad (32)$$

$$Phosphorous\ stresss = 200 * \frac{\frac{Actual\ phosphorous\ content}{Optimal\ phosphorous\ content}}{(\frac{Actual\ phosphorous\ content}{Optimal\ phosphorous\ content} + \exp\left(4.065 - 0.0535 * \frac{Actual\ phosphorous\ content}{Optimal\ phosphorous\ content}\right))} \quad (33)$$

2.5. Calculating Yield, Water and Nutrient Requirements

The equations from the previous sections have been implemented in in MATLAB®. The following figure shows a graph of the progression of daily biomass for different crops over their growing period.

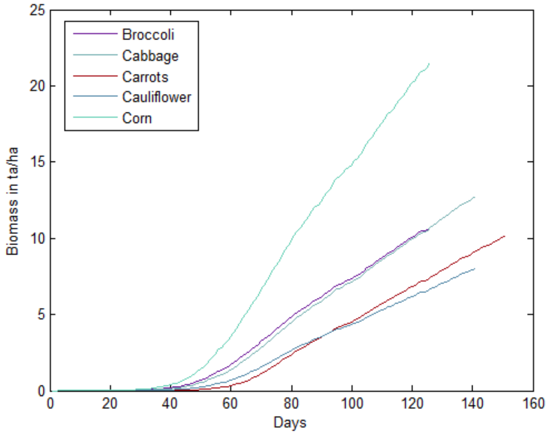

Figure 2: Biomass accumulation

The model can also be used to calculate the daily irrigation requirements for different crops. The daily water requirement is the volume of water necessary per hectare to keep the water stress value to 1. The evapotranspiration model calculates the amount of water lost by the crop each day. Daily precipitation is responsible for making up for this lost water. In the absence of precipitation there needs to be water supplied through irrigation to make up for this water deficit. The model calculates the amount of water required each day, in liters, to make sure there is a stress free growth.

Figure 3 shows the amount of water required for different amounts of rainfall. The model calculates the water required to make sure that there is no water stress. The graph shows that the water required decreases as the amount of rainfall increases. A similar analysis can be done to show the water requirements for different soil types under the same weather conditions.

Similar to water use, the model also calculates the required fertilizer, in tonnes, to make sure that there is no nitrogen and phosphorous stress on the crops. Fertilizers are defined by their rating, which is the percentage of nitrogen and phosphorous content in the given fertilizer. For example, a fertilizer with a 35-40 rating contains 35% of nitrogen and 40% of P2O5. For a 100 pound bag, this would mean a nitrogen content of 35 pounds, and 40 pounds of P2O5. To calculate the amount of fertilizer required, we simply have to divide the amount of nitrogen, or phosphorous required by the percentage rating. If a crop requires 10 pounds of phosphorous per day, the farmer would need to apply 10/0.4 = 25 pounds of fertilizer. Figure 4 shows the fertilizer required for broccoli for, in kg/ha, for the various planting start dates.

3. OPTIMAL CROP SCHEDULER

The objective of this model is to select the list of crops and their planting dates to maximize the potential yield. Since each crop has a yield on a different scale, we will need to use a more normalized measure to measure the yield. For example, a yield of 10 tonnes/ha might be a poor return for a potato crop, while a yield of 5 tonnes/ha might be very good for an eggplant crop. This problem can be overcome by first simulating the crop yields for all the possible planting dates. The yield at each planting date is then divided by the maximum yield for that crop. This serves to provide an accurate measure of performance for the crop by showing its proximity to the maximum potential yield.

The only constraint on the model is to ensure that at any given time more than one crop cannot be planted. The model is also setup to ensure that there is a 1 month fallow period after planting each crop.

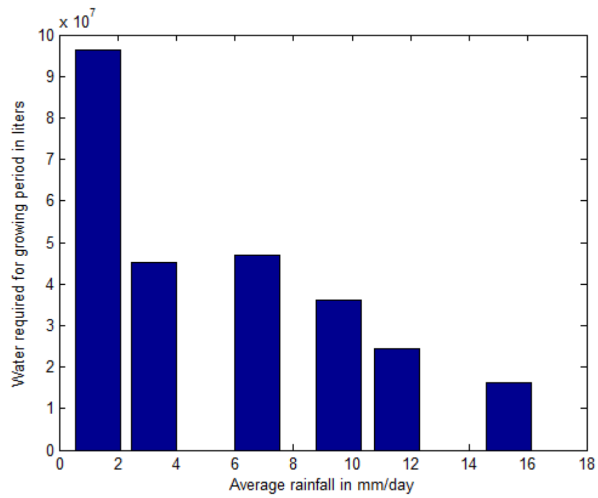

Figure 3: Water requirements for different average rainfall levels

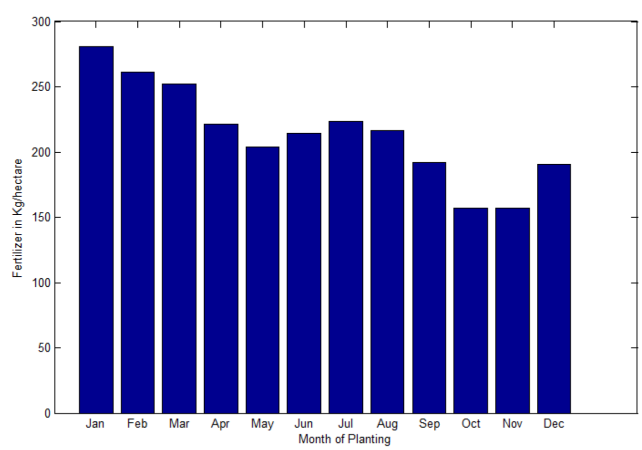

Figure 4: Fertilizer required in Kg/hectare for various planting dates

There is no crop rotation constraint, and the same crop can be planted in succession. The model is written as a binary integer program, where each decision variable can only take the values 0 or 1. The yields have been calculated for the beginning of each month during a 4 year simulation cycle. The list of variables is as follows:

- Y_{ij} : Normalized yield of crop j when planted on planting date i.
- x_{ij} : Decision variable for crop j on planting date i. It is a binary decision variable with 0 and 1 as the only possible variables.
- totaltime : Length of the simulation in months.
- numcrops : Total number of crops available for simulation.
- $\mathrm{cropduration_j}$: Duration of crop cycle for crop j

$$maximize \sum_{i=1}^{totaltime} Y_{ij}x_{ij} \quad \forall j = 1,2, \ldots .number\ of\ crops \tag{34}$$

$$\sum_{i=1}^{totaltime}\left(x_{ij} + \sum_{k=1}^{numcrops}\sum_{l=1}^{cropduration_j} x_{k(i-l-1)}\right) \le 1$$

$$\forall j = 1,2, \ldots .number\ of\ crops \tag{35}$$

The model was simulated for 48 months using weather data from http://globalweather.tamu.edu/. The weather data was for a tropical region with average temperatures of around 25^{0}C, and an average rainfall of 13cm. The model can currently optimize 28 crops. To visualize the results of the simulation a Gantt chart has been used. This chart is useful to display the start date and the duration of the crops. An initial run for all the 28 crops was made to determine the most optimal sequence of crops. Figure 5 displays the results from this simulation. The figure shows that broccoli, potatoes, barley, oats and millet produced the most optimal yield.

This model can now be extended to account for crop rotation. Since the decision variables are months, it is not possible to explicitly implement crop rotation for the crops. Crop rotation is implemented by specifying that the same crop not be re-planted for a minimum of three months. If the other crops do not fit the solution, the same crop can be planted again after waiting for 3 months. A new variable called *rotation* is added to the model. This variable specifies the duration for which a crop cannot be reused on the same field. This value can take a value between 1-3 months.

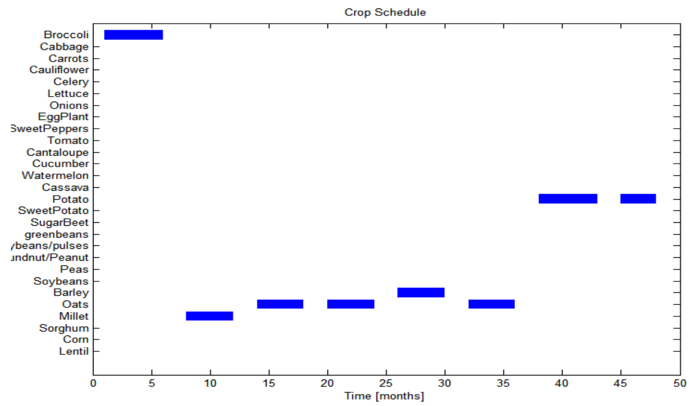

Figure 5: Crop scheduling for no crop rotation constraints for the full set of crops.

Using the same objective function as eq. 34, the new model constraint can be written as:

$$\sum_{i=1}^{totaltime}\left(x_{ij} + \sum_{m=i+cropduration_j}^{i+cropduration_j+rotation} x_{mj} + \sum_{k=1}^{numcrops}\sum_{l=1}^{cropduration_j} x_{k(i-l)}]\right) \le 1$$

$$\forall j = 1,2, \ldots .number\ of\ crops \tag{36}$$

The objective function used in all these models has been aimed at selecting crops that have the best performance. In addition to this objective we can introduce three new objectives. The first objective is aimed at increasing economic output. To do this we will need to multiply the prices of the various crops to the yield. These prices have been obtained from the USDA website. The data covers a range of 48 months from the beginning of 2009 to the end of 2012. Figure 6 shows the price($)/pound of three crops overs this range.

The objective function for maximizing the economic impact for regular planting duration and truncated duration is given as:

$$maximize \sum_{i=1}^{totaltime} C_{ij}Y_{ij}x_{ij}$$

$$\forall j = 1,2, \ldots .number\ of\ crops \tag{37}$$

$$maximize \sum_{i=1}^{totaltime} C_{ij}\left(Y_{ij}x_{ij} + Y'_{ij}x'_{ij}\right)$$

$$\forall j = 1,2, \ldots .number\ of\ crops \tag{38}$$

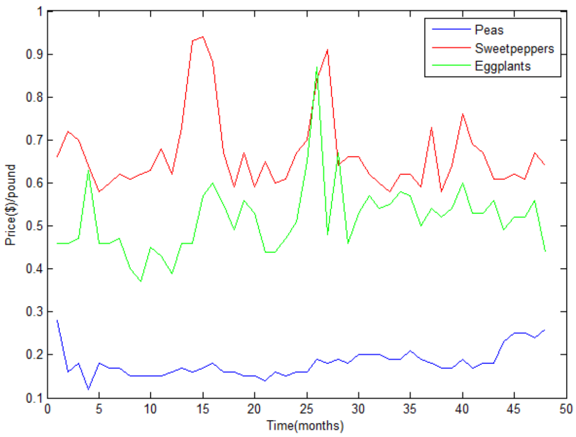

Figure 6: Price($)/pound for 48 month period

C_{ij} is the price/pound of crop i in the month j. The constraints for the three configurations are the same as the previous model. The next objective is decreasing the environmental impact of the vegetables selected. The model aims to select those crops that consume the least amount of water. The crop growth model gives us an estimate for the amount of water required for a certain crop for its growth period. Figure 7 shows the water requirements for three crops over a 48 month duration.

The objective function for minimizing the environmental impact for regular growing period and truncated period is given as:

$$minimize \sum_{i=1}^{totaltime} W_{ij}x_{ij} \quad \forall j = 1,2, \dots .number\ of\ crops \qquad (39)$$

$$minimize \sum_{i=1}^{totaltime} \left(W_{ij}x_{ij} + W'_{ij}x'_{ij}\right) \quad \forall j = 1,2, \dots .number\ of\ crops \qquad (40)$$

W_{ij} and W'_{ij} are the water requirements for crop i when planted in month j for a full growing period and a truncated growing period respectively. The constraints for the three configurations are the same as before.

Finally, we also have a combined objective of maximizing the economic output while minimizing the environmental impact. To achieve this we will multiply the water quantity requirements of the crops with a monetary value P_j and subtract it from the economic objective. The objective functions for the regular and truncated growing periods are given as:

$$maximize \sum_{i=1}^{totaltime} \left(C_{ij}Y_{ij}x_{ij} - P_jW_{ij}x_{ij}\right) \quad \forall j = 1,2, \dots .number\ of\ crops \qquad (41)$$

$$maximize \sum_{i=1}^{totaltime} \left(C_{ij}Y_{ij}x_{ij} - P_jW_{ij}x_{ij}\right) + \left(C_{ij}Y'_{ij}x'_{ij} - P_jW'_{ij}x'_{ij}\right) \quad \forall j = 1,2, \dots .number\ of\ crops \qquad (42)$$

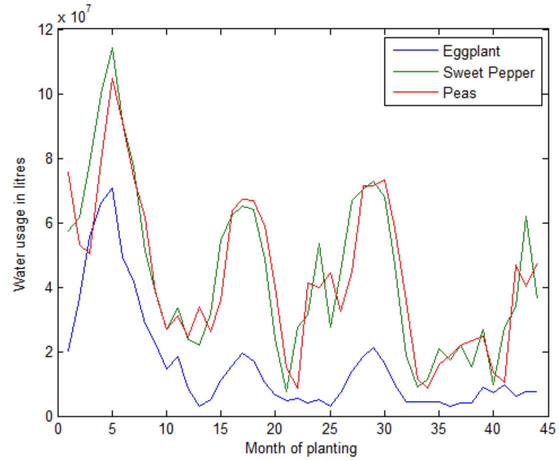

Figure 7: Water requirements in litres for a 48 month period

The values for the water price P_j are uniformly distributed between $0.0005-0.001 per liter.

4. RESULTS

This section presents the results for the 4 objectives discussed in the previous section. Each objective has 2 configurations; no crop rotation and explicit crop rotation of 3 months. Figure 8 present the results for a set of three crops: peas, sweet peppers and egg plants. These crops were chosen because they had yields in comparable ranges. From the figure we can see the change in schedules for the different objectives and configurations.

Sweet peppers and eggplant are dominant for economic and environmental objectives respectively. Peas are dominant for the performance objective. In a multi objective scenario we can see a healthy mix of all three crops being selected. We can also observe the change in scheduling for the various configurations across the different objectives.

5. CONCLUSIONS AND FUTURE WORK

In this paper we have studied the behavior of mathematical programming models for selecting and scheduling the planting of crops to fulfill various objectives. These objectives cover issues like performance, economic viability, and environmental impact but are not exhaustive. The decision variables in the developed model only simulate crop selection and scheduling. Decisions like resource utilization, borrowing money, farming practices and abandonment of farms need to be implemented to simulate real world situations.

The economic viability objective needs to include real world features like risk, market fluctuations and resource contentions. Similarly, the environmental impact objective only considers water usage. Future iterations of these models should also model impact of fertilizers, condition of soil and contamination of immediate environment. The models used in this chapter are binary integer programming models. This makes the model very time consuming to run. When we modeled the performance objective using all the crops, the model ran for over five hours to produce a solution. This issue can be addressed by turning the problem into a quadratic objective programming, or using alternative algorithms to find a solution. We can also use linear programming models to implement mixed cropping schemes.

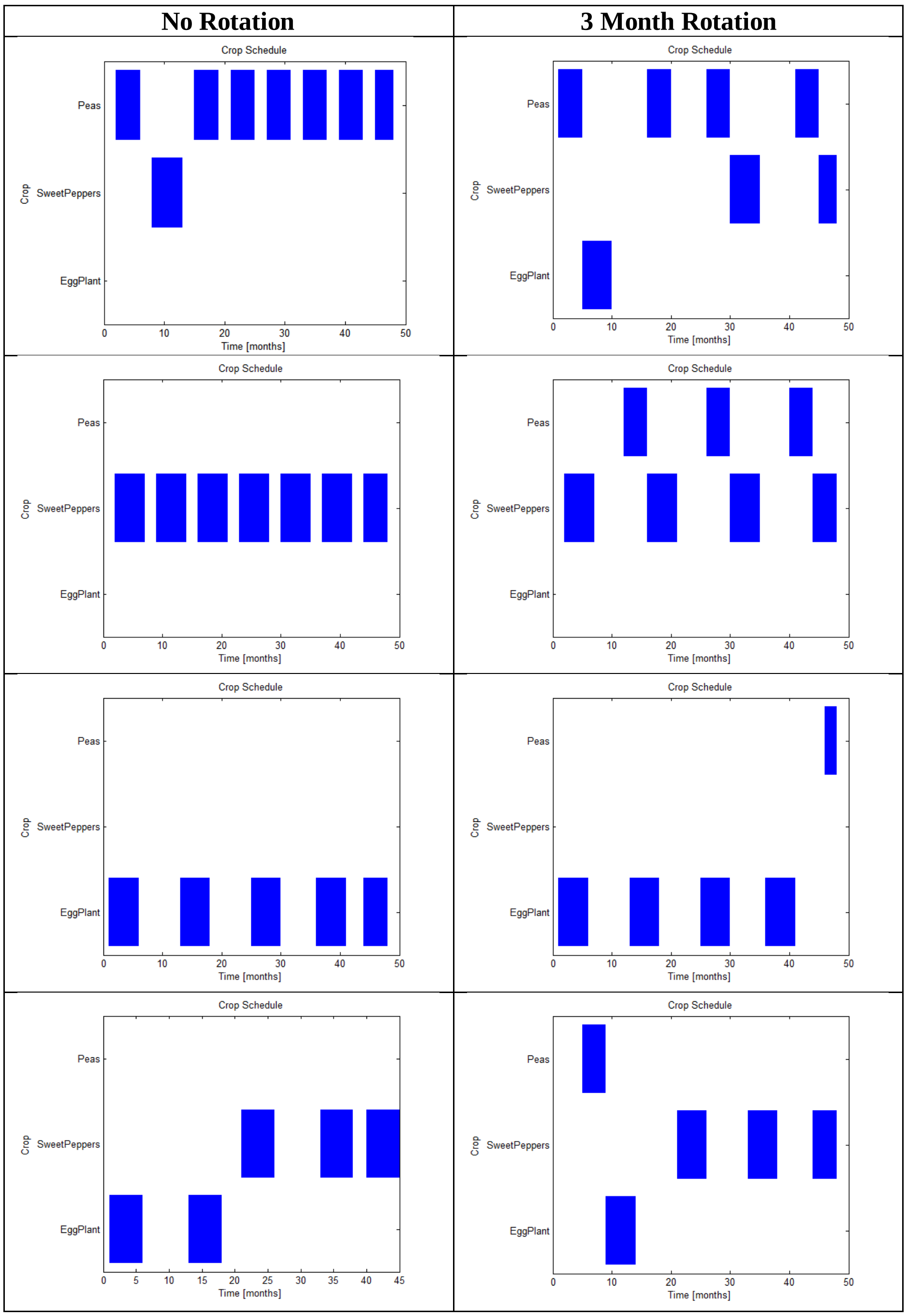

Figure 8: Scheduling of crops for different objectives. Row1-performance. Row2-Economic output. Row3-Environmental impact. Row4-Multi objective.

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