AHP-K-GDSS: A NEW SORTING METHOD BASED ON AHP FOR GROUP DECISIONS

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ABSTRACT

Some public buildings need for energy requalification intervention as they are responsible for a significant share of energy consumption and other related CO2 emissions. With tight budget constraints choices have to be made.
To solve this problem a group sorting decision support system based on the analytic hierarchy process, the K-means algorithm has been developed. The system aims at sorting alternatives into ordered classes of importance. A case study carried out in an Italian municipality allowed us to verify the validity of our new method in a real setting.

Keywords: Energy requalification; decision support system; AHP; clustering; sorting

1. INTRODUCTION

The Analytic Hierarchy Process (AHP) is a multi-criteria decision-making method developed by Saaty in the 1970s (Saaty 1977). It has been widely used for ranking a finite set of alternatives and for choosing the best alternative from a finite set of alternatives (Ishizaka and Labib 2014). In the paper (Ishizaka, Nemery and al. 2012), AHP was adapted in AHPSort in order to deal also with sorting problems.

A sorting problem aims to assign each alternative into one of the predefined ordered classes (Ishizaka and Nemery 2013b). In the case of problems with a large set of alternatives, AHPSort enables us to avoid the construction of a pairwise comparison matrix including all the alternatives. The alternatives are not compared with each other but only with the profiles representing the classes. Thus, the pairwise comparison matrix is much smaller. In the case of problems where the set of alternatives could change (by either adding or removing an alternative), using AHPSort can avoid modifying the pairwise comparison matrix of the alternatives and recalculating the priorities.
When an alternative is removed, its attached pairwise comparison matrix is also removed but the other pairwise comparison matrices are untouched. When an alternative is added, a new pairwise comparison matrix is added and only the pairwise comparisons of the alternative with the profiles representing the classes need to be provided.
However, sometimes the decision-maker is unable to provide the reference profiles. In this case, Lolli, Ishizaka et al. (2014) have developed Analytic Hierarchy Process-K (AHP-K). It is a hybrid method based on AHP for the evaluation of the weights and the K-means for the sorting of alternatives into K-ordered classes. This paper adapts this multi-criteria decision sorting method used by single decision-maker to group decisions. The aim of the resulting Analytic Hierarchy Process-K-Group Decision Support System (AHP-K-GDSS) is to incorporate all the actors' opinions, to mitigate their subjectivity and to evaluate objective and subjective criteria in one model. This new methodology was developed to solve a real case study of energy requalification of pebble buildings and utilities. The rest of the paper is presented as follow. Section 2 contains the literature review. Section 3 presents the new methodology. Section 4 describes the case study and section 5 concludes the paper.

2. LITERATURE REVIEW

Sorting problems came to the attention of researchers and practitioners later than ranking problems. AHP (Saaty 1977), PROMETHEE (Brans and Vincke 1985), MACBETH (Bana e Costa, De Corte et al. 2012), TOPSIS (Lai, Liu et al. 1994) and ELECTRE III (Roy 1978) are some popular multi-criteria approaches that have been extended to sorting problems, respectively leading to AHP-Sort (Ishizaka, Nemery et al. 2012), AHP-Sort II (Miccoli and Ishizaka 2017), GAHPSort (López and Ishizaka 2017), FlowSort (Nemery and Lamboray 2008), GAIA-Sort (Nemery, Ishizaka et al. 2012), MACBETHSort (Choudhary and Shankar 2012), TOPSIS-Sort (Sabokbar, Hosseini et al. 2016), ELECTRE-Tri (Yu 1992) and its variant ELECTRESort (Ishizaka and Nemery 2014), ELECTRE Tri-C (Almeida-Dias, Figueira et al. 2010)(Almeida-Dias, Figueira et al. 2010), ELECTRE Tri-nC (Almeida-Dias, Figueira et al. 2012), ELECTRE Tri-nB (Fernández, Figueira et al. 2017). Most sorting approaches assume a single DM or a group acting as one (Ka 2011), and the

goal of finding an agreed classification method is therefore neglected in the case of multiple DMs. However, most decisions are taken by several decision-makers (Ishizaka and Nemery 2013). Multiple criteria sorting in the context of group decision-making is a challenging field of research (Gothwal and Saha 2015), with applications to several operational settings.
Two recent multi-criteria decision-making approaches address group sorting problems. The first one was proposed by Wang and Chen (2006), based on intuitive fuzzy outranking relations among the alternatives. The second one (Lolli, Ishizaka et al. 2015) represents the extension of FlowSort (Nemery and Lamboray 2008) to group decisions. This paper present an extension to group decision for AHPSort-K.

3. METHODOLOGY

The developed Group Decision Support System (GDSS) consists of two stages. The first stage (Lolli, Ishizaka et al. 2016) involves sorting alternatives into ordered classes (Lolli, Ishizaka et al. 2016). The second stage involves selecting the alternatives to be funded under the constraints of a limited budget.

3.1. Sorting stage with AHP-K-GDSS

Step 1: Pair-wise comparison between criteria
The *I* criteria are pair-wise compared in order to calculate their relative weights w_i^k with the eingevalue method (Ishizaka 2014), where $i = 1, \dots, I$, is the criterion and $k = 1, \dots, K$ is the decision-maker. The K priority vectors are thus obtained as the eigenvectors associated with the highest eigenvalues for each comparison matrix. Consistency analysis must be performed (Saaty 1980) to ensure the judgmental consistency of the matrices, that is to say compliance with the transitivity rule in terms of a consistency ratio lower than a typical threshold value of 0.1.

Step 2: Score assignment
Each alternative is evaluated directly (e.g. assignment of a score on a scale 1-20) or indirectly (e.g. with pairwise comparison matrices) on each criterion.

Step 3: Individual Ranking
The universal priority of alternative a_j for decision-maker k is thus obtained by the weighted sum of $v_{i,j}^k$:

$$P_j^k = \sum_{i=1}^{I} w_i^k \times v_{i,j}^k \tag{1}$$

where $j = 1, \dots, J$ and $k = 1, \dots, K$

Let $\overline{P_J^k}$ be the ordered array of P_j^k, i.e. the individual ranking for decision-maker j. Hence, different K rankings are now available.

Step 4: Universal ranking
The weights of the decision-makers K (i.e. u^k) are assigned by the owner of the decision-making process such that $\sum_{k=1}^{K} u^k = 1$ in accordance with their experiences, skills, etc. The universal ranking is obtained by a further weighted sum of P_j^k (1) as follows:

$$P_j = \sum_{k=1}^{K} u^k \times P_j^k \qquad \text{with } j = 1, \dots, J \tag{2}$$

where $\overline{P_J}$ is the ordered vector of P_j, i.e. the universal ranking.

Step 5: Universal Sorting
In order to sort the alternatives into N classes, the K-Means algorithm is applied in the universal ranking of Step 4.
The aim is to create compact and well-separated N classes of alternatives for $\overline{P_J}$ by minimising the sum of the squared distances between the centroid of each class (i.e. the mean point in the case of Euclidean distance) and the items in the class. This step allows all the alternatives to be classified into classes with different degrees of priority.

Step 6: Individual Sorting for the Veto application
This optional further step aims to compare the universal sorting with the individual sorting. In order to find the individual sorting, the K-Means algorithm is now applied to $\overline{P_J^k}$, where $k = 1, \dots, K$.

An alternative a_j may be classified into a universal class C_n, very distant from the individual class C_n^k of a particular DM. This divergent opinion can now be taken into account with a veto. That is to say, the alternative is downgraded or upgraded in order to classify it into a universal class closer to the individual one. In sum, the veto applied to the DMs avoids the full compensation of the weighted sum (Step 4) by opportunely modifying the final sort on the basis of the individual sort.

4. CASE STUDY

4.1. Introduction

The proposed decision support system was validated in an Italian municipality of about 30,000 inhabitants. Energy is becoming a precious resource (Ishizaka, Siraj et al. 2016). The aim of the study was to choose between 34 interventions of energy requalification $j = 1, \dots, 34$
Three council members served as decision-makers, with the mayor leading the decision process. In particular, DM1 is a budget representative, DM2 is responsible for social policies and DM3 is an environmental expert.
The mayor assigned weights to the DMs on the basis of their experience and skills. In this case, the weights were set equal, i.e. $u^1=u^2=u^3=0.33$. The decision-makers were asked to directly assign scores on a scale of 1-20 to the alternatives as regards to the criteria

4.2. Criteria

A brainstorming performed by the decision-makers gathered the following criteria: Annual CO2 savings (c1), Annual monetary savings (c2), Financial payback time (c3), Comfort improvement (c4), Image toward citizens (c5), Educational value (c6), Local employment development (c7), Increase in energy self-sufficiency (c8). Each DM pairwise compare them and the derived weights are given in the table 1.

Table 1: Weights of the criteria

	DM1	DM2	DM3
c1	0.071	0.054	0.459
c2	0.231	0.023	0.12
c3	0.331	0.018	0.082
c4	0.048	0.153	0.056
c5	0.024	0.385	0.02
c6	0.033	0.223	0.028
c7	0.157	0.105	0.038
c8	0.106	0.038	0.197
CR	0.03	0.09	0.06

4.3. Results

The individual rankings (Step 3) were achieved by multiplying the scores by the weights of the criteria (Table 1) for each DM. The global rankings were then obtained by aggregating these individual rankings through a weighted sum over DMs (Step 4), with DMs equally weighted in this case.

The K-means algorithm was launched on the global ranking in order to achieve the global sorting into three ordered clusters named C1, C2 and C3 from the most to least preferred (Step 5). As the K-means is a greedy algorithm, the final partition depends on both the number of iterations and the starting partition. In order to verify the robustness of the solution, the algorithm was launched 20 times with 100 iterations, each time starting from a different random partition. It has been found that the solution does not change over launches, and can therefore be considered robust. In order to apply the veto system, the global sorting approach has to be compared with the individual sorting, which was also obtained with the K-means algorithm (Step 5). Table 3 shows the global rankings of the alternatives, with global priorities reported in brackets, along with the global sorting. Furthermore, the last column the final classes after the veto application (Step 6), where the symbol "-" indicates that no change in sorting occurred. Since the number of classes and decision-makers is three in this case, the veto is simply expressed by the following two conditions: i) if an alternative is globally sorted as C1 (C3), but at least one DM classifies it as C3 (C1), then it is reclassified into C2; and ii) if an alternative is globally sorted as C2, but two DMs classify it as C3 (C1), then it is reclassified into C3 (C1). In our case, only the alternative 17 was reclassified into C2 for condition i).

Table 1: Sorting of the alternatives

Alternatives	*Global ranking and priority*	*Global sorting*	*Veto*
1	30 (7.73)	C3	-
2	34 (7.10)	C3	-
3	18 (8.71)	C3	-
4	24 (8.14)	C3	-
5	33 (7.48)	C3	-
6	25 (8.03)	C3	-
7	31 (7.70)	C3	-
8	26 (7.88)	C3	-
9	27 (7.78)	C3	-
10	21 (8.50)	C3	-
11	19 (8.54)	C3	-
12	20 (8.53)	C3	-
13	22 (8.49)	C3	-
14	9 (10.24)	C2	-
15	7 (10.31)	C2	-
16	11 (9.96)	C3	-
17	1 (12.32)	C1	C2
18	16 (8.94)	C3	-
19	29 (7.74)	C3	-
20	17 (8.91)	C3	-
21	12 (9.90)	C3	-
22	14 (9.45)	C3	-
23	10 (10.01)	C3	-
24	5 (11.36)	C2	-
25	28 (7.75)	C3	-
26	32 (7.69)	C3	-
27	3 (11.69)	C2	-
28	2 (12.10)	C1	-
29	4 (11.68)	C2	-
30	23 (8.41)	C3	-
31	8 (10.27)	C2	-
32	6 (10.60)	C2	-
33	13 (9.58)	C3	-
34	15 (9.35)	C3	-

The alternative in C1 will be funded. The alternatives in C2 will be funded depending on their global ranking and funds available. The alternatives in C3 will not be funded.

5. CONCLUSIONS

A new AHP-based group sorting method has been defined with the aim of classifying a set of alternatives into a predefined number of ordered classes, without recourse to limiting profiles defined by decision-makers. They were simply asked to assign scores to the alternatives, while the k-means clustering algorithm was used to classify the alternatives into well-separated clusters. This automatically solves a sorting problem when decision-makers are not confident in providing the limiting profiles, and the generation of a predefined number of not-empty classes has to be forced. The individual sorting was also achieved with the aim of allowing the application of a veto system. This is the first AHP-based group sorting approach that has been developed.

The application of this group decision support system to a real case study has validated the approach and confirmed its usefulness in real settings, where transparent procedures are required in decision-making groups.

Topics for further research may include distance-based models for veto application, as well as fuzzy or evidence-based theories for dealing with the uncertainty of decision-makers' judgements. An index of compactness to measure the quality of the sorting could also be developed.

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