TRANSFER AND GENERALISATION OF FINANCIAL RISK METRICS TO DISCRETE EVENT SIMULATION

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ABSTRACT
Quantitative Finance is one of the numerous application fields of discrete event simulation. Because of special requirements in this area, typically domain specific simulation tools are applied, instead of general purpose simulators. It appears fruitful and beneficial to provide some of the risk metrics common in quantitative finance for discrete event simulation in general, in order to make use of them in generalised versions in further domains. In this paper we describe transfer and generalisation of risk metrics from quantitative finance to general purpose simulators with regard to Semi-Variance, Value at Risk, Expected Shortfall and Drawdown.

Keywords: risk metrics, discrete event simulation

1. INTRODUCTION

The field of Quantitative Finance (also called *Computational Finance* or *Financial Engineering*) deals with computer-supported analysis of price histories of asset values and the support of investment decisions in financial markets. Next to Monte-Carlo-Simulation (i.e. mathematical method, that solves complex problems from probability theory numerically, based on repetitive random experiments following the Law of large Numbers, see Metropolis and Ulam (1949)) and related methods, discrete event simulation is mainly applied in two areas:

- On the one hand, simulating the performance of financial markets on micro level, i.e. down to the level of single market participants (Arthur, Holland, LeBaron, Palmer 1997; Lux and Marchesi 2000; Levy, Levy, and Solomon 2000; Hommes 2006; LeBaron 2006).
- On the other hand, evaluating particular financial market trading strategies by simulating, assessing and optimising them in different historical market environments (Chande 1997, Kocur 1999, van Tharp 2007).

In the context of this paper, we focus on the second application area.

For the evaluation of trading strategies, special purpose simulators are applied, so-called *back tester*s. Back testers differ from general purpose simulators in the following aspects:

1. Instead of common random number generators, historical time series are used as data sources for security prices.
2. Entities in the sense of classical simulation objects are not required, as only the behaviour of defined trading strategies in the context of inflowing market data is analysed. From a conceptual point of view, these strategies do not necessarily have to be represented as entities.
3. Waiting queues and higher modelling components such as processing stations or transport stations are not explicitly required for modelling, due to the immaterial nature of financial strategies and their market orders. Likewise, synchronization mechanisms for different entities are usually not needed.
4. However, there are extensive requirements on the characterisation of trading strategies, in particular related to profitability and the risk taken. Here, computation of manifold special key figures developed in quantitative finance is required for an extended reporting. To our knowledge, most of these key figures and their underlying concepts have not been regarded in general purpose simulation so far.

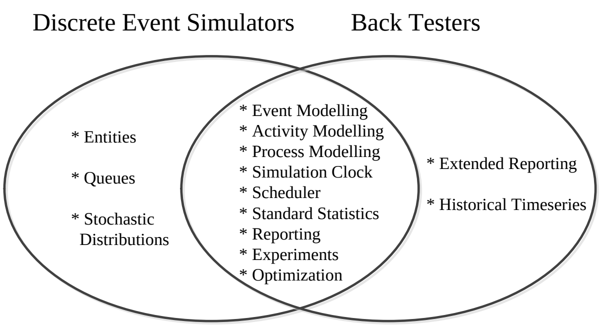

Figure 1: Commonalities and differences of general purpose discrete event simulators and back testers

Commonalities and differences between general purpose discrete event simulators and back testers are shown in the figure above.

In spite of the differences mentioned, back testers and general purpose simulators are widely comparable in structural terms. Further, the modelling and simulation cycle as well as experiments are processed equivalently. Back testers can be understood as a special case of general purpose discrete event simulators and therefore implemented by these, see e.g. Golombek (2010) or Koors and Page (2011).

From a historical point of view, back testers have developed concurrently to general purpose simulators since the nineties, with rather limited mutual exchange of ideas into both directions.

Risk metrics are an advanced aspect of back testers, both serving for quantification of risk of a particular trading strategy and for comparisons of different trading strategies amongst each other. To us, risk metrics do appear potentially useful for other application domains as well.

In this paper, we aim at the transfer and generalisation of established risk metrics from quantitative finance into the world of general purpose discrete event simulators.

This paper is structured as follows: In section 2, we deal with the character of risk, seen from the quantitative finance point of view. Parallels to application fields of simulation are shown. We advance to the concept of downside risk and motivate that the idea of transferring financial risk metrics to general application domains of simulation could be beneficial. In section 3, we describe four central risk metrics of quantitative finance in their original context first, and then illustrate them by simulation queues. Advancing to observation variables, we generalise the concepts and transfer them into the field of general purpose discrete event simulation. We outline the modifications and enhancements we have carried out and discuss certain implementation aspects. Section 4 summarises and concludes the paper.

2. THE CONCEPT OF RISK IN QUANTITATIVE FINANCE

2.1. Expected Value as Characteristic and Variance as Risk

Yield and risk are the central concepts when evaluating financial trading strategies by means of back testers.

Here, *yield* is understood as expected value of the return of a trading strategy during a defined time span. The trading strategy may carry out a number of investment decisions during the simulated time frame, so-called *trades*. The compounded return of all single trades is the *overall return* of the strategy at the end of the simulation. Its expected value is the yield of the strategy.

The second central characteristic of trading strategies is risk. Risk is defined as *volatility*, i.e. variation of return around the expected average return, following the fundamental thought pattern called *Mean-Variance-Framework* introduced by Markowitz (1952) into finance. Volatility mathematically corresponds to the standard deviation of return.

General purpose simulation of discrete event systems operates with mean and empirical standard deviation as well, e.g. regarding queue length or concerning the state space of observation variables in general.

Attention should be paid to a shift in connotation of the aforementioned concepts in finance: While the expected value of return is considered as given and characteristic for a strategy, variance always has a negative connotation, in the sense of risk.

From this point of view, an expected queue length x of a standard M/M/1-queuing system would be considered merely a characteristic of the system. With increasing variance of queue length (at a constant expected value) the model would be estimated increasingly *risky*, in the sense of higher uncertainty and precariousness.

In this sense, risk can be understood as a metric for the potential of a strategy or a model to leave a stable equilibrium state into an undesired direction.

In many typical application fields of simulation, the departure from an equilibrium state or from an interval of tolerable states is also seen as critical, e.g. in

- Queuing systems and production systems, if queues run empty and machine utilisation sinks towards 0, resp. conversely, if the available waiting room capacity is exceeded and therefore client orders are lost
- Ecological systems, if necessary population sizes or quantities of substance are fallen below or exceeded, and the system collapses
- Physical systems, if material strains are too high, resulting in damages. Physiological systems may suffer from underutilisation as well, thus becoming inoperative in consequence of non-use.

2.2. Downside Risk as Asymmetric Risk Conception

Deviations from the mean may be uncritical into one direction, while undesired into the other direction. In finance, only below-average returns (resp. above-average losses) pose a risk for an investment, while excess returns are welcome and may be ignored in terms of risk assessment. Quantitative finance has elaborated an asymmetrical risk metrics category called *downside risk*, where only one-sided variations of return in the sense of underperformance are considered as risk.

Asymmetric risk perceptions can also be found in the application fields of simulation, with regard to desired resp. undesired deviations from means or system equilibrium states. Thus longer queues in production, higher pollutant concentrations in ecological systems or stronger physical strains will generally be considered as more risky and less desirable, while this is usually not true for the opposite cases. On this background of comparable asymmetric evaluation preferences, downside risk metrics from quantitative finance should be more suitable for risk assessment in simulation application fields than conventional symmetric standard statistics.

2.3. Practical Implementation

We would like to provide modellers of general discrete event systems with additional tools, which allow them to assess inherent "risks" of models more adequately, following the concepts of quantitative finance. This can help in understanding model dynamics more appropriately and can deliver new fruitful approaches and deeper insight concerning analysis and adaptation of undesired model behaviour.

For this purpose, the four risk metrics from quantitative finance discussed below are transferred into our general purpose discrete event simulation framework *Desmo-J* (www.desmo-j.de, Page and Kreutzer 2005) as statistical extensions. This work is currently carried out in the context of a bachelor thesis in our working group Modelling and Simulation (MBS) in the Department of Informatics at University of Hamburg.

3. RISK METRICS

A risk metric is a concept to assess risk. In comparison, a risk measure is the implementation of a computational process, employed to calculate a certain risk measurement. As we focus on the conceptual side of risk, the term *risk metric* is used in this paper.

In this section we describe four central risk metrics of quantitative finance in their original context first and afterwards exemplarily illustrate them by simulation queues. Advancing to observation variables, we generalise the concepts and transfer them into the field of general purpose discrete event simulation. We outline the modifications and enhancements we have carried out and discuss certain implementation aspects.

Formal definitions of the mentioned risk metrics in their original financial context can be found in e.g. Yang, Yu, and Zhang (2009); Lohre, Neumann, and Winterfeldt (2009) or Giorgi (2002).

3.1. Semi-Variance

As stated above, only those trades yielding a below-average return actually contribute to the downside risk of a trading strategy. By contrast, trades with above-average returns are welcome and do not increase downside risk. Insofar, only those undesired return deviations below expected return are accounted for in the Semi-Variance concept. The computation is carried out as for the standard variance, but observations above the mean are skipped.

In the characterization of a queue, we can as well assume that only one of the two possible deviation directions from the mean queue length is preferable, depending on the context. This means that in computing Semi-Variance only time spans are to be considered, where the average queue length is exceeded or fallen below, respectively, depending on the preferred point of view. (Commonly, there will be a preference for shorter queue lengths.)

By this metric, we can gain a first impression and a basis for comparison, with regard to the size of undesired variations of queue length.

For the implementation of further risk metrics described below, it is required to store all single observations as time series, until the simulation has ended. Thus the implementation of Semi-Variance accesses the total sample collected at the end of a simulation run, in contrast to the stepwise online computation of standard statistics, as normally applied in *Desmo-J* (Page, Lechler, and Claassen 2000).

In order to provide general applicability concerning the direction of deviation perceived as risk, we compute negative as well as positive Semi-Variance and provide both of them separately on simulation reports.

3.2. Value at Risk

The *Value at Risk* (VaR) of an open trade quantifies the maximum loss (in absolute currency units) that will not be exceeded at a given confidence level of $1 - \alpha$, at the end of a set period. In other words, VaR is equivalent to the α-quantile of the probability distribution of the returns expected in the set period.

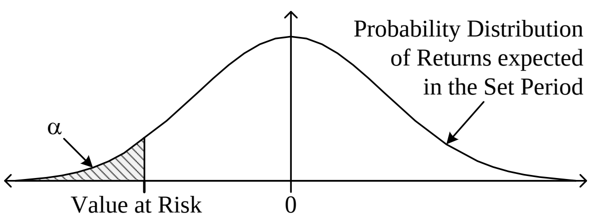

Figure 2: Illustration of the Value at Risk metric

The basic return distribution for computation can be determined by Historical Simulations, Monte-Carlo-Simulation or the Variance-Covariance method (Linsmeier and Pearson 2000).

Value at Risk is an important key figure in banking: Under the Basel II accord, banks are legally obligated to compute market risk in terms of the Value at Risk metric on a daily basis, in order to ensure that pre-set maximum losses won't be exceeded within certain time horizons.

Transferred to queues in general purpose discrete event simulators, VaR indicates the minimum or maximum queue length expected after a given simulation time interval and at a set confidence level, starting from the current queue length. This becomes an important measurand, if certain queue lengths must not be exceeded or fallen below, e.g. because of cost restraints or capacity limits of the waiting room. Thus, VaR gives a formative indication how to dimension a waiting room at a given initial state, a set confidence level and a designated time horizon, in order to meet specific restrictions.

In simulation practice, it should be avoided to refer to the current queue length resp. the present value of an observation variable, as these values permanently change during simulation runs and therefore are not eligible as fixed states of reference for the VaR measure. Instead, we implement the VaR concept by only considering the relative change of observation variables compared to their previous states and call this *Delta at Risk*.

In case of bounded state spaces, there is a risk of distortion at boundary states and extreme states, such as the length of a queue cannot fall below zero. In this context, no further decrease of the queue length will be observable next. In contrast, if the state of an observation variable is far away from boundary states and extreme states, their impact on the next observations will be much smaller.

Without differentiation, this could lead to overestimating the risk of *increase* of queue length in cases of lengths > 0, as more length increment observations starting from length = 0 would be regarded than appropriate in the normal case.

Conversely, observations in the context of a queue length > 0 would distort the representative basis of future states for length = 0, as unrealistic length decrement observations were included in the sample, though for length = 0 a decrease of queue length below zero is conceptually impossible.

The stated danger of reduced significance due to insufficient consideration of marginal or extreme contexts exists in the practical use of Value at Risk in financial institutions as well. Often the present state of financial markets is abstracted from, and the risk of loss is calculated without consideration of the current context. For example, the risk of high losses intuitively is lower at the end of a financial market decline than at the beginning of the same period, as most fearful investors have already left the market at an earlier stage, thus selling pressure eases. Nevertheless, the current market environment normally is not considered when calculating VaR.

As long as state change probabilities are determined regardless of the context of boundary states and extreme states, the VaR metric consequently runs into danger of diminished significance.

We address this problem by calculating four different Delta at Risks in simulation reports, according to four contexts: On the one hand we determine the Delta at Risk related to the most frequent and the median state of all states observed during the simulation run. On the other hand we compute two more Delta at Risk measures, corresponding to the minimum and maximum states observed. Using the example of queues, output is generated for the expected alteration of queue length considering empty, frequent, median and maximal length queues.

The choice of the median state in the sense of a representative average state is motivated by the approach to analyse a state as far away from boundary states and extreme states as possible, in order to provide a largely unaffected Delta at Risk representing intermediate states.

In case of non-symmetrical state distributions, the most frequent state is situated closer to boundary states or extreme states than the median state. Even though it might be under (partial) influence of boundary states and extreme states, the most frequent state may be regarded as a better basis for significant conclusions in certain contexts, as statements concerning this state may have higher empirical correspondence.

The description above deals with the original and probably most frequent application of VaR as a risk metric for one-dimensional discrete state spaces (here: currency units). In principle, the Delta at Risk concept is canonically extendable to multi-dimensional or continuous state spaces as well. In order to keep simulation reports manageable, the mapping of sets of multi-dimensional states or intervals of states to a one-dimensional discrete state space should be considered, though.

With this in mind, we continue to describe Delta at Risk in terms of one-dimensional discrete state spaces, as we expect this to be the most common use case.

A naive implementation of Delta at Risk of an observation variable could compute and store the delta of state size divided by the simulation time passed since the last state change, as a quotient, at every change of the observation variable. By sorting these rates of change in ascending order, accumulating their frequencies and normalising these, the distribution function F(x) could be constructed, describing the distribution of rates of change per reference time unit.

However, rates of change computed in this way would base on variable length time intervals containing only one actual change event, being scaled to a reference time unit afterwards. Real consecutive observations within real reference time intervals would be ignored. Thus, extrapolations of short time intervals could lead to excessive distortions when dealing with longer time intervals.

Instead, we determine and store the size of state change at every modification of an observation variable, as compared to the state the variable had a fixed time interval earlier. For this purpose the state history for (at least) the time interval under consideration has to be stored within a time series during the simulation run.

Subsequently, the recordings of these actual relative state changes within the set time span, are sorted in ascending order, accumulated and normalised in frequency, yielding to a more realistic distribution function F(x). This procedure takes into account that subsequent state changes may neutralise each other partially or entirely over longer time frames, as often observed in practice.

For flexibility, *n* time spans of interest may be passed as input parameters, leading to a risk analysis for each of the time frames given, regarding the cumulative outcome of all multiple state changes actually observed within that time frame, recorded at every simulation event concerning the observation variable.

Beyond the original application in quantitative finance, we extend the initially asymmetrical concept of downside risk to both ends of the state space, as it cannot be assumed that risk always is represented at the left end of the state space. Thus, we appraise the potential risk at the right end of the distribution likewise. As a consequence the α-quantiles for α = 1%, 2.5%, 5%, 10% as well as for 90%, 95%, 97.5% and 99% are determined from $F(\alpha)$ and output on the simulation report.

In sum, the Delta at Risk metric derived from Value at Risk quantifies the maximum size of change expected (i.e. risk, in terms of quantitative finance) with regard to an observation variable, at a given confidence level α, after a set period, and according to four well-defined reference states.

Typical conclusions based on the simulation report were "At a confidence level of 97.5% and starting from the observed median *m*, the queue length will maximally increase by *x* entities and maximally decrease by *y* entities after 10 minutes of simulated wall clock time" or "Starting with an empty queue and given a confidence level of 99%, the queue length will not exceed *z* entities after 1 hour of simulated wall clock time".

3.3. Expected Shortfall

Value at Risk quantifies the maximum loss at a given confidence level of $1 - \alpha$, nevertheless a loss exceeding VaR is not impossible, as long as $\alpha > 0$. The shortcoming of the VaR concept is that it does not make a statement about the amount of loss to be expected, if the limit of Value at Risk is exceeded in critical cases.

This gap is filled by the metric *Expected Shortfall* (also referred to as *Conditional Value at Risk* or *Expected Tail Loss*, Rockafellar and Uryasev 2000). It expresses the expected amount of loss for the α fraction of cases where VaR is exceeded. Hence, Expected Shortfall is a metric to assess the potential extent of damage for unlikely but possible cases of extreme events (in terms of the choice of α). Expected Shortfall is an important key figure used to describe the state space beyond VaR when structuring finance products with insurance nature.

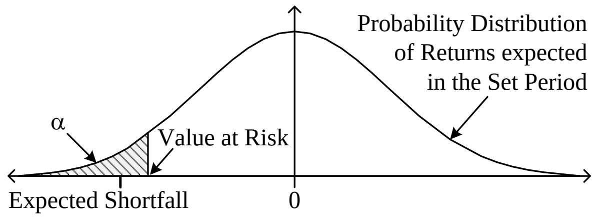

Figure 3: Illustration of the Expected Shortfall metric

Here too, we generalise the quantitative finance metric with regard to three aspects, for the purpose of transferring the concept to general simulation application domains: Firstly, we move from expected absolute loss to expected relative state change of an observation variable. Secondly, we consider minimum, median, most frequent and maximum states as references. Thirdly, both ends of the probability distribution are regarded likewise, to remain flexible with respect to where to attribute risk, depending on the special application area.

In order to avoid confusion, the modified risk metric is called *Conditional Delta at Risk*.

Conditional Delta at Risk is based on the same data as Delta at Risk introduced above. In the course of calculating Delta at Risk, the Conditional Delta at Risk simply can be computed as the expected value of the empirical probability density below (resp. above) the α-quantile of all observations.

Referring to queues, Conditional Delta at Risk indicates the expected growth or contraction of queue length for the remaining α fraction of cases beyond the confidence level. If a waiting room was dimensioned taking account of the Delta at Risk metric, its overload in the remaining α fraction of cases is now appraisable.

A typical conclusion based on the simulation report would be "If, starting with an empty queue and given a confidence level of 99%, the queue length exceeds the Delta at Risk of *z* entities after 1 hour of simulated wall clock time, then an average queue length of *z* + *c* entities can be expected".

3.4. Drawdown Phases

The term *Drawdown* of a trading strategy relates to an interim loss of asset value, after a new peak of asset value was reached beforehand. Drawdown may be given in absolute currency units or as a percentage of the preceding peak asset value. A *Drawdown Phase* often extends over several consecutive (mis-)trades and thus cumulates their effects.

Drawdown Recovery starts at the point of maximum interim loss. It lasts until the previous peak asset value is reached again or exceeded.

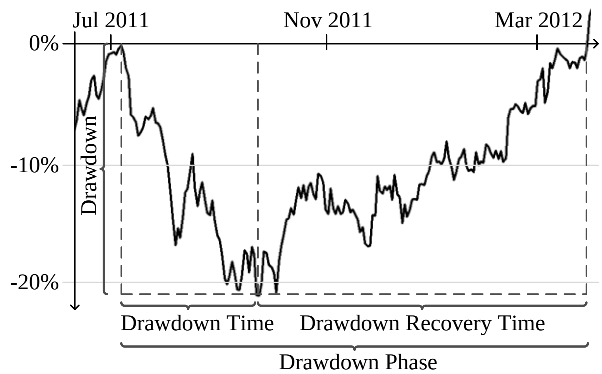

Figure 4: Drawdown Phase of the L'Oréal Share from July 2011 to April 2012

Drawdown and *Drawdown Time* give an impression of extent and speed at which the state of observation variables may move into an undesirable direction. Therefore, these key figures allow an assessment of undesirable system dynamics in terms of vulnerability or susceptibility to disturbances. By contrast, the *Drawdown Recovery Time* provides an indication of the regenerative capacity of the analysed system.

A financial trading strategy may experience a multitude of Drawdown Phases over time. Especially the *Maximum Drawdown* ever undergone is of particular interest with regard to trading futures contracts in financial markets, as this key figure determines the required minimum margin of a trading account, to withstand the highest Drawdown encountered so far in the strategies history.

Drawdowns and their recoveries can only be quantified ex post, when a Drawdown Phase is completed and a new peak in asset value has been reached. Moreover, a trading strategy is almost always in a Drawdown, except from new peaks in asset value. Hence it is of vital interest to analyse the structure of Drawdowns to gain insight into the dynamics of undesirable behaviour.

In the context of queues, a queue length of 0 may be set as base level, corresponding to the peak asset value in financial context. Then Drawdown, Drawdown Time and Recovery Time characterise the dynamics of formation and reduction of queues, regarding the queue length as observation variable.

Since a multitude of Drawdown Phases per observation variable is to be expected in simulation runs, we extend the quantitative finance Drawdown concept, as it originally focuses only on the extreme case Maximum Drawdown resp. Average Drawdown.

To provide a quick overview of the total dynamics of the system modelled, we classify all Drawdowns according to their absolute extent and display their distribution in a histogram. The number of histogram bins is determined according to the rule of Freedman and Diaconis (1981), after the simulation has ended.

Two additional histograms visualise the distribution of Drawdown Times and Drawdown Recovery Times in a similar manner.

For further orientation, we introduce a Drawdown scatter plot, encoding Drawdown Time as *x*-coordinate, Drawdown Recovery Time as *y*-coordinate and the Drawdown extent as colour of a data point. Hereby character and distribution of all Drawdown Phases during the simulation run can be seen at a single glance.

Moreover, all Drawdown Pathways per observation variable are superimposed in a joint coordinate system. Thus, a good overview of the typical and most severe Drawdown Phases is given, including the Recovery sub-phases.

A second diagram visualises the superimposed time series only of the Recovery sub-phases per observation variable, providing a quick overview of the regenerative properties of the system modelled.

Beyond the specified extended analysis of the Drawdown concept itself, we generalise this risk metric in three ways, in order to support its flexible and unrestricted utilisation in simulation application domains:

- We consider both the setbacks and recoveries on the way towards peak states (“classical” Drawdowns) and complementarily the ascent and descent phases on the way towards bottom states. By this means, we again take into account that the interpretation of a certain state development direction as risky or preferable cannot be predetermined for the manifold application areas of simulation.
- Furthermore, after determination of the median state at the end of a simulation run, the time series of observed variable states is divided into phases below and above the median. These phases are treated separately as Drawdowns and Recoveries concerning states below the median resp. as ascents and descents concerning states above the median. This supports the alternative point of view of striving for a central state of equilibrium and considering deviations from this balanced state as risk. Since phases below and above the median are treated separately, it remains free whether risk is attributed to one or both directions of deviation.
- For non-symmetrical empirical distributions, the same handling as above is applied, but this time with reference to the state with the highest frequency instead of the median state.

Accordingly, all advanced statistical and graphical analysis mentioned (3 histograms, 1 scatter plot, 2 time series diagrams) are provided for all six use cases of the generalised Drawdown concept described above.

4. SUMMARY

In quantitative finance, specialized discrete event simulators called *back testers* are utilized, in order to evaluate financial market trading strategies. Here, strategies are simulated in different historical market environments and evaluated, compared and optimised by means of a wide range of assessment criteria. A significant assessment category is related to the *risk* taken in following a particular trading strategy. In this context, risk in terms of *volatility* is understood as a metric for the potential to deviate from a characteristic average rate of return. Additionally, quantitative finance has elaborated the concept of downside risk in the form of asymmetrical risk metrics, where only negative deviations in the sense of underperformance are regarded.

We propose to introduce the four most accepted financial risk metrics of back testers into general purpose discrete event simulators. We think that these metrics open up new and fruitful views on model dynamics in general and may specifically support the evaluation and possibly optimisation of undesired model behaviour. In particular, dimensioning of waiting rooms as well as planning of processing capacities should benefit from the generalised risk key figures.

In order to support a preferably wide field of application domains in discrete event simulation, we extend the transferred metrics *Value at Risk* and *Expected Shortfall* in three aspects: Firstly, we advance from expected absolute loss of currency units to expected relative changes of observation variables, to allow deriving general statements independently from particular current states. Secondly, we consider the minimum, median, most frequent and maximum state of observation variables in order to handle boundary and extreme states separately. In this sense, the median of an observation variable represents a state as far as possible from extreme situations. For non-symmetrical empirical distributions, the most frequent state is regarded as well, as a maybe better basis for significant conclusions. Thirdly, we account for both ends of state distributions, since depending on the application domain, risk may be regarded as deviation into different directions, possibly also into both directions.

The third aforementioned generalisation is also applied when transferring *Semi-Variance* to discrete event simulation.

The second and third extension mentioned above concern *Drawdown Phases*, too.

We aim at providing a concrete tool for the modeller of general discrete event models, in order to convey an impression of the value of transferring quantitative finance risk metrics into other domains. For this reason, our general purpose simulation framework *Desmo-J* is extended by these concepts in a Bachelor thesis at the working group of Modelling and Simulation in the Department of Informatics at University of Hamburg. We expect to provide a more sophisticated risk estimation in the various application domains of discrete event simulation as compared to conventional standard statistics.

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