SYNTHESIS OF ADEQUATE MATHEMATICAL DESCRIPTION AND ITS APPLICATIONS

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ABSTRACT
The problems of the synthesis of an adequate mathematical description and his further use are considered. The definition of an adequate mathematical description of the physical process was given. In paper an analysis of methods for constructing such descriptions were performed. Algorithm of the synthesis of adequate mathematical description was described in the frame of one method. For the case when the parameters of the mathematical model are given approximately a few of non-standard synthesis problems of adequate mathematical description were formulated. Some results of the construction of such descriptions are given in the references. The problems of use the results of mathematical simulation for constructing of reliable forecast were discussed. One of possible algorithms of obtaining of reliable results of forecasting using the adequate mathematical description is suggested.

Keywords: adequate mathematical description, reliable prediction, non-standard synthesis problems.

1. INTRODUCTION

The main objective of mathematical simulation of the behavior of dynamic systems and physical processes is an coincidence of results of mathematical modeling of selected mathematical description of the real process with the experimental data. Under the mathematical description of the physical process is understood analytical relationship (differential, algebraic, integrated, etc.) of defined structure between the selected state variables of the system under study (mathematical model) and external influences (load). Naturally the structure of the mathematical model, the number of state variables, the coefficients may be differently and they are determined by objectives of study of particular physical of the process.

If the results of mathematical modeling do not match with experimental data, then further use of these results is problematic. Consequently such a coincidence is the sufficient and necessary conditions for the successful use of mathematical simulation in practice.

An important concept in this regard is the adequacy of the constructed mathematical description of the physical process under study. Let us give the possible definition of such description.

2. DEFINITION OF ADEQUATE MATHEMATICAL DESCRIPTION

Definition. Mathematical description will be called the local stable adequate mathematical description (ALSMD) of the investigated process if the results of mathematical simulation using this description coincide with the experimental data with an accuracy of measurements in a small neighborhood of the initial data; parameters of ALSMD resistant to small changes of the input data.

The comparison of the results of mathematical modeling with experimental data in determining ALSMD ensures objectivity of the results of mathematical modeling.

In the given work the mathematical models of physical processes described only by the linear system of the ordinary differential equations will be examined. Such idealization of real processes or dynamic systems is widely used in various areas for the description of control systems, as well as of mechanical systems with the concentrated parameters (Stepanov, Shorkin and Gordon 2004; Lino, Maione 2008; Pop, Valle and Cottron 2004; Krasovskij 1968; Sarmar, Malik 2008; Porter 1970; as well as economic processes (Laitinen 2013), biological and ecological processes (Alexik 2000; Thomas 2004) etc. It is shown that with the help of such systems in some works even human emotions is simulated (Breitenecker, et al 2009). This, of course limits the class of the physical processes. However, all the features of the solution of this problem are also valid for most other types of physical processes.

We assume that the mathematical model of the physical process is represented in the form:

$$\dot{x}(t) = \tilde{C}\, x(t) + \tilde{D}\, z(t)\,, \qquad (1)$$

with the observation equation

$$y(t) = \tilde{F}\, x(t)\,, \qquad (2)$$

where $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T$ is vector-function variables characterized the state of process, $z(t) = (z_1(t), z_2(t), ..., z_l(t))^T$ is vector-function of unknown external loads, $y(t) = (y_1(t), y_2(t), ..., y_m(t))^T$; $\tilde{C}, \tilde{D}, \tilde{F}$ are matrices of the appropriate dimension with constant coefficients which are given approximately, $\tilde{F}$ is nonsingular matrix dimension $m \times n$ and rang $\tilde{F} = n$; X, Y, Z – complete valued function spaces; $(.)^{T}$ is a mark of transposition. We assume that the matrix $\tilde{F}$ is the identity matrix $\tilde{F} = E$, then $m = n$ for simplicity.

There are two main approaches for the synthesis of an adequate mathematical description using experimental measurements (Stepashko 2008; Gubarev 2008; Gukov 2008; Menshikov 2009):

- a priori selected a particular mathematical model of the physical process and its structure (matrixes $\tilde{C}, \tilde{D}, \tilde{F}$) and then selected such model of external load which together with the chosen mathematical process gives the results of mathematical modelling which coincide with experiment with precision of measurements (Menshikov 2008, 2009a, 2009b);
- Some models of external loads are given a priori and then mathematical model of process of type (1) is chosen for which the results of mathematical simulation coincide with experiment (Stepashko 2008; Gubarev 2008; Gukov 2008).

Now we will consider the synthesis of adequate mathematical description in the frame of first approach analyzing the process with the concentrated parameters, for which the motion is described by ordinary differential equations of n-order (1) (Menshikov 2008, 2009a, 2009b).

The problem of constructing an *adequate mathematical description* of physical processes that are described by the mathematical model (1), (2) can be formulated as follows: it is necessary to find an unknown vector function $z(t) = (z_1(t), z_2(t), \ldots, z_l(t))^*$ so that the vector function $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^*$, which is obtained from the system (1), (2), coincides with the experimental data $\tilde{y}(t) = (\tilde{y}_1(t), \tilde{y}_2(t), \ldots, \tilde{y}_m(t))^*$ with an accuracy of obtaining experimental measurements in the selected functional metric.

In this definition of adequate mathematical description there are a number of shortcomings which do not allow you to uniquely identify an adequate description among of all possible. The first of these shortcomings is the lack of ambiguity in the case if not all state variables are measured (the matrix is not the identity or matrix is a degenerate, etc.). The coincidence of part of the state variables with the experimental data does not guarantee the coincidence of the remaining state variables with experiment. In addition, the deviation of the results of mathematical simulation of experimental data is determined by the properties of the function spaces X, Z, U , as well as the type of norms defined in these spaces.

Of great importance is also the value of $[\,a, b\,]$ of independent variable. If the size of this interval is increased several times then an adequate mathematical description may lose its properties. Therefore, the above definition of an adequate mathematical description gives only the basic necessary requirements, but does not give a clear definition of an adequate description.

Let us consider questions what prospects of adequate mathematical descriptions are valid for further use and what goals should be selected at the creation of adequate mathematical descriptions.

It will be useful to address to classical works in this area. In work (Shanon 1975) the following statement was done: "…the goal of the imitation simulation is the creation of experimental and applied methodology which aimed at the use of it's for a prediction of the future behaviour of system".

So the mathematical simulation using the adequate mathematical descriptions first of all are aimed at the forecast of behaviour of real processes. With the help of mathematical cimulation (iclude of adequate mathematical description) must have the possiblity to predict of behaviour of real process in new conditions of operation. For example, it is possible to test more intensive mode of operations of the real machine without risk of its destruction. Such tool (right mathematical description) allows to simulate the characteristics of process in the unconventional modes of operations, and also to determine optimum parameters of real process.

The considered situation requires the formation of some uniform methodological approach to this problem, creation of general algorithms and common criteria of adequacy evaluation (Menshikov 2008, 2009a, 2009b).

3. STATEMENT OF THE SYNTHESIS PROBLEMS

In (Menshikov 2008, 2009a, 2009b) proposed an algorithm for constructing an adequate mathematical description in frame of the first approach. The problem is reduced to the solution of several integral equations of the first kind:

$$A_{p,i}\, z_i = B_{p,i}\, x,\ i = 1,2,...,l \qquad (3)$$

where $A_{p,i}$ are integral completely continuous operators; $A_{p,i}: Z \to U$; $B_{p,i}: X \to U$ are continuous linear operators; $z_i(t)$ is i-th component of the vector of unknown function $z(t) = (z_1(t), z_2(t), \ldots, z_l(t))^*$; $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^*$ is a vector function of the state variables of the system (1); Z, X, U there are some normalized functional spaces. Operators $A_{p,i}, B_{p,i}$ depend continuously on the vector parameter $p = (p_1, p_2, \ldots, p_q)^*$ mathematical model of the physical process (the coefficients of the matrices $\tilde{C}, \tilde{D}$).

If the part of external loads of real process is known, this case can be reduced to one which is examined earlier with using the linearity of initial dynamic system (1).

We assume that state variables $x_i(t), 1 \le i \le n$ of system (1) correspond to some real characteristics $\tilde{x}_i(t), 1 \le i \le n$ of physical process which are investigated and that the vector function $\tilde{y}(t) = \tilde{F}\,\tilde{x}(t)$ was obtained from experimental measurements and presented by graphics $x_{\delta_0}(t) = (\tilde{x}_1(t), \tilde{x}_2(t), \ldots, \tilde{x}_n(t))^*$. Due to the measurement error, we assume that the inequality is valid

$$\left\| x_{ex} - x_{\delta_0} \right\|_X \le \delta_0, \tag{4}$$

where δ_0 there is a given value; $x_{\text{ex}}(t) = (x_{1,ex}(t), x_{2,ex}(t), \ldots, x_{n,ex}(t))^*$ is the exact function of the vector of state variables.

Then equation (3) can be rewritten as

$$A_{p,i}\, z_i = B_{p,i}\, x_\delta = u_{\delta_i},\ i = 1,2,\ldots,l. \tag{5}$$

Let us consider the set of possible solutions $Q_{\delta,p,i}$ of the equation (5), taking into account only the error in the initial data x_{δ_0}:

$$Q_{\delta_i,p} = \left\{ z \in Z : \left\| A_{p,i} z - B_{p,i} x_{\delta_0} \right\| \le \left\| B_{p,i} \right\|_{X \to U} \delta_o = \delta_i \right\}. \quad (6)$$

Due to the fact that the operators $A_{p,i}$ are completely continuous, the set $Q_{\delta_i,p}$ is unlimited for any i, p, δ in norm of Z (Tikhonov, Arsenin 1975).

Suppose $\Omega[z]$ is some positive continuous functional which is defined on a normed function space Z.

To obtain the stable solutions of ill-posed problem (5) we can use the variation principle of their construction proposed by (Tikhonov, Arsenin 1975).

Consider now the following extreme problem:

$$\Omega[z_{\delta,p}] = \inf_{z \in Q_{\delta,p}} \Omega[z], \tag{7}$$

where functional $\Omega[z]$ has been defined on set Z (Tikhonov, Arsenin 1975).

Theorem. If the functional space Z is reflex Banach space, the functional $\Omega[z]$ is convex and lower semi-continuity on Z, Lebesgue set $Q_{\delta_i,p}$ $Z_d = \{z : \Omega[z] \le \Omega[z_0]\}$ for some function from $Q_{\delta,p}$ is bounded then the function $z_{\delta,p} \in Q_{\delta,p}$ exists.

It was shown that under certain conditions the solution of the extreme problem (7) exists, is unique and stable with respect to small change of initial data u_δ (Tikhonov, Arsenin 1975).

Note that there is no sense to investigate behaviour of the obtained solution at $\delta \to 0$.

4. THE SYNTHESIS PROBLEMS FOR CLASS OPERATORS

Consider a situation where adequate and sustainable mathematical description of the physical process (for fixed parameters of the mathematical model and its structure) has already been constructed using any method.

What practical value will have this description in the prediction of the physical process, considering that the parameters of the original mathematical model and its structure is given approximately with a given accuracy?

So, it is supposed that the vector parameters p is given inexactly. So vector p can get different values in given closed domain $D: p \in D \subset R^N$.

Some operators $A_{p,i}, B_{p,i}$ correspond to each vector from D. The sets of possible operators $A_{p,i}, B_{p,i}$ have been denoted as classes of operators $K_A = \{A_{p,i}\}, K_B = \{B_{p,i}\}$. So we have $A_{p,i} \in K_A, B_{p,i} \in K_B$. The deviations of operators $A_{p,i} \in K_A$ between themselves from set K_A and operators $B_{p,i} \in K_B$ from set K_B are given:

$$\sup_{p_\alpha,\, p_\beta \in D} \left\| A_{p_\alpha,i} - A_{p_\beta,i} \right\| \le h_1,\ \sup_{p_\gamma,\, p_\lambda \in D} \left\| B_{p_\gamma,i} - B_{p_\lambda,i} \right\| \le d_1.$$

Now we transfer to consideration of a more general problem of synthesis of external loads functions in which the inaccuracy of operators $A_{p,i}, B_{p,i}$ will be taken into account.

The set of possible solution of equation (6) is necessary to extend to set $Q_{h_1,d_1,\delta}$ taking into account the inaccuracy of the operators $A_{p,i}, B_{p,i}, p \in D$ (Tikhonov, Arsenin 1975):

$$Q_{h_1,d_1,\delta} = \{z : \| A_{p,i} z - B_{p,i} x_\delta \| \le h_1 \| z \|_U + B_0 \},$$

where $B_0 = d_1 \| x_g \| + \| B_{p,i} \| \delta$.

Any function from $Q_{h_1,d_1,\delta}$ causes the response of mathematical model coinciding with the response of investigated object with an error into which errors of experimental measurements and errors of a possible deviation of parameters of a vector $p \in D$ are included. A problem of a finding $z \in Q_{h_1,d_1,\delta}$ will be entitled by analogy to the previous one as a *problem of synthesis for a class of operators (*Menshikov 2008, 2009b).

It should be noted that the set of the solutions of a problem of synthesis $Q_{h_1,d_1,\delta}$ at the fixed operators $A_{p,i} \in K_A, B_{p,i} \in K_B$ in $Q_{h_1,d_1,\delta}$ contain elements with unlimited norm (incorrect problem) therefore the value $(h_1 \|z\|_U + B_0 \cdot \delta)$ can be infinitely large. Formally speaking, such situation is unacceptable as it means that the error of mathematical simulation tends to infinity, if for the simulation of external load is used the arbitrary function from $Q_{h_1,d_1,\delta}$ as functions of external load. Hence not all functions from $Q_{h_1,d_1,\delta}$ will serve as "good" functions of external load.

The function of external loads $z(t)$ in this case can be different. They will depend on final goals of mathematical simulation. For example, we can obtain model z_{p_0}, z_{p_1} for simulation of given motion of system as solution of extreme problems (Menshikov 2011, 2013):

$$\Omega[z_{p_0}] = \inf_{p \in D} \inf_{z \in Q_{\delta,p} \cap Z_1} \Omega[z], \; p \in R^N, \tag{8}$$

$$\Omega[z_{p_1}] = \sup_{p \in D} \inf_{z \in Q_{\delta,p} \cap Z_1} \Omega[z], \; p \in R^N. \tag{9}$$

Model z_{p_0} together with the matrices $\tilde{C}, \tilde{D}$ (coefficients of which are determined by the vector of parameters $p_0 \in D$) provide an adequate mathematical description of the physical process. If the functional $\Omega[z]$ characterizes energy of the external control a physical process, then the result is the solution of the extreme problem (8) gives the optimal adequate mathematical description with minimum energy.

Model z_{p_1} together with the matrices $\tilde{C}, \tilde{D}$ (coefficients of which are determined by the vector parameters $p_1 \in D$) provide an adequate mathematical description of the physical process. If the functional $\Omega[z]$ characterizes energy of the external control a physical process, then the result is the solution of the extreme problem (9) gives the optimal adequate mathematical description with maximum energy.

The function of external loads which is necessary for estimation from below of output of dynamic system (process) can be obtained as solution of the following extreme problem (Menshikov 2008, 2009a, 2009b):

$$\| A_{p_b} z_b \| = \inf_{p \in D} \| A_p z_{\delta,p} \|. \tag{10}$$

where $z_{\delta,p}$ is the solution of extreme problem (7).

Another model for estimation from above of output of dynamic system (process) can be obtained as solution the extreme problem (Menshikov 2011, 2013):

$$\| A_{p_c} z_c \| = \sup_{p \in D} \| A_p z_{\delta,p} \|. \tag{11}$$

As unitary model z_{un} we can call the solution of following extreme problem (Menshikov 2008, 2009a, 2009b):

$$\| A_{p_{un}} z_{un} - B_{p_{un}} x_\delta \| = \inf_{p \in D} \sup_{p \in D} \| A_p z_{\delta,a} - B_p x_\delta \|_U \tag{12}$$

where $z_{\delta,a}$ is the solution of extreme problem (7) with $p = a, a \in D$.

The triple $\{A_{p_{un}}, B_{p_{un}}, z_{un}\}$ gives the stable adequate mathematical description for class of operators of process as example of possible one. Function z_{un} provides a minimum deviation of the state variables of the mathematical model of the physical process under the worst combination of parameters (coefficients of the matrices $\tilde{C}, \tilde{D}$).

The method of special mathematical model selection was suggested to increase of approximate solution exactness of extreme problems (8) – (12).

The problems of synthesis of steady adequate mathematical models of special use are investigated. The examples are given (Menshikov 2011, 2013).

5. POSSIBLE USE OF ADEQUATE MATHEMATICAL DESCRIPTION FOR FORECAST

Suppose that an ALSMD has already been constructed, using some approach. For further use of it for the purpose of predicting the behavior of a physical process, it is important to have confidence in the fact that the description in the new conditions will remain adequate. According to the definition given above, the check-up of adequacy of the mathematical description can be performed only in the presence of experimental measurements, which correspond to the new conditions. But then it defeats the purpose of predicting the behavior of a physical process by means of mathematical simulation. Thus, in regime of forecasting cannot be verified adequacy in principle.

To justify the plausibility (reliability) forecasting in the new environment may use the property of inertia of dynamical systems: small changes in the initial data correspond to small changes in the behavior characteristics of dynamic systems. Therefore, if you use an ALSMD in the new conditions, which vary only slightly, then the results of mathematical simulation will be a little different from the previous results, for which experimental data are available. But in this case also defeats the purpose of mathematical simulation.

Similar results will be obtained if we consider that an ALSMD is robust to small changes in the initial data. For example, if the model of external load (which is part of an ALSMD) is robust to small changes in the initial data (matrixes and vector function), then in the new close conditions the results of mathematical simulation will differ little from the future experiments. In this case, the results of mathematical simulation will not have a scientific interest as they will be almost the same as the previously conducted mathematical simulation and experiment.

Here is proposed the following algorithm for the use of mathematical modeling to build of reliable forecast: for each of several small neighborhoods of change the parameters of the physical process is synthesized its ALSMD further options of ALSMD extrapolated (or interpolated) to the new neighborhood change the parameters of the physical process and conducted mathematical modeling in the new environment.

Suppose that the researcher is interested in the behavior of the physical process in fundamentally new conditions, for example, in a significant increase in the temperature of the process.

In this case, even an ALSMD will not give reliable results predict the behavior of the physical process in the new environment. Let under the temperature t_1 an ALSMD (function $z_1(t)$ for given matrices $\tilde{A}, \tilde{C}, \tilde{F}$ in equation (1)) was obtained.

Then the experimental measurements of the state variables of the mathematical model (1) are carried out for considerable increase of temperature t_2. And in this case a new ALSMD (new function $z_2(t)$ for a given matrixes $\tilde{C}, \tilde{D}$ in equation (1)) will be constructed.

Further analysis of changes in the functions $z_1(t)$, $z_2(t)$ (it is important that ALSMD is stable to small changes of the initial data) when the temperature of the physical process changes from t_1 till t_2. These changes in the function $z(t)$ extrapolated to the new function $z_3(t)$ for temperature t_3 (when $t_1 < t_2 < t_3$) or make interpolation to a new function $z_3(t)$ for the temperature t_3 (when $t_1 < t_3 < t_2$). Using for mathematical simulation of the physical process a new function $z_3(t)$ for fixed matrices $\tilde{C}, \tilde{D}$ we can obtain reliable results of mathematical simulation. It is obvious that such an approach should have reliable information about the continuous change of the parameters of the physical process by change of temperature from t_1 to t_3. This information can only be obtained from physical experiments and observations. It is necessary to be sure that the physical process does not change qualitatively. For example, the object does not melt at the new temperature t_3.

Mathematical description of physical process do not checked for adequacy before simulation and after it. Therefore mathematical simulation is often compared with art (Shanon 1975).

This situation greatly reduces the usefulness of mathematical simulation (sometimes to negative values). This happens in cases where the results of mathematical simulation are contrary to the experimental measurements.

So it is necessary to have the unified methodological approach to solving this problem, creation of synthesis algorithms and formation of evaluation criteria of reliability of the results of mathematical simulation (Menshikov 2008, 2009a, 2009b).

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